

## **Radiative Properties of Cirrus Clouds in the Infrared (8-13 $\mu\text{m}$ ) Spectral Region**

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## Abstract

Atmospheric radiation in the infrared (IR) 8-13  $\mu\text{m}$  spectral region contains a wealth of information that is very useful for the retrieval of ice cloud properties from aircraft or space-borne measurements. To provide the scattering and absorption properties of nonspherical ice crystals that are fundamental to the IR retrieval implementation, we use the finite-difference time domain (FDTD) method to solve for the extinction efficiency, single-scattering albedo, and the asymmetry parameter of the phase function for ice crystals smaller than 40  $\mu\text{m}$ . For particles larger than this size, the improved geometric optics method (IGOM) can be employed to calculate the asymmetry parameter with an acceptable accuracy, provided that we properly account for the inhomogeneity of the refracted wave due to strong absorption inside the ice particle. A combination of the results computed from the two methods provides the asymmetry parameter for the entire practical range of particle sizes between 1  $\mu\text{m}$  and 10000  $\mu\text{m}$  over wavelengths ranging from 8  $\mu\text{m}$  to 13  $\mu\text{m}$ . For the extinction and absorption efficiency calculations, several methods including the IGOM, Mie solution for equivalent spheres (MSFES), and the anomalous diffraction theory (ADT) can lead to a substantial discontinuity in comparison with the FDTD solutions for particle sizes on the order of 40  $\mu\text{m}$ . To overcome this difficulty, we have developed a novel approach called the stretched scattering potential method (SSPM). For the IR 8-13  $\mu\text{m}$  spectral region, we show that SSPM is a more accurate approximation than ADT, MSFES, and IGOM. The SSPM solution can be further refined numerically. Through a combination of the FDTD and SSPM, we have computed the extinction and absorption efficiency for hexagonal ice crystals with sizes ranging from 1 to 10000  $\mu\text{m}$  at 12 wavelengths between 8 and 13  $\mu\text{m}$ .

Calculations of the cirrus bulk scattering and absorption properties are performed for 30 size distributions obtained from various field campaigns for midlatitude and tropical cirrus cloud systems. Parameterization of these bulk scattering properties is carried out by using second-order polynomial functions for the extinction efficiency and the single-scattering albedo and the power law expression for the asymmetry parameter. We note that the volume-normalized extinction coefficient can be separated into two parts: one is inversely proportional to effective size and is independent of wavelength, and the other is the wavelength-dependent effective extinction efficiency. Unlike conventional parameterization efforts, the present parameterization scheme is more accurate because only the latter part of the volume-normalized extinction coefficient is approximated in terms of an analytical expression. After averaging over size distribution, the single-scattering albedo is shown to decrease with an increase in effective size for wavelengths shorter than 10.0  $\mu\text{m}$  whereas the opposite behavior is observed for longer wavelengths. The variation of the asymmetry parameter as a function of effective size is substantial when the effective size is smaller than 50  $\mu\text{m}$ . For effective sizes larger than 100  $\mu\text{m}$ , the asymmetry parameter approaches its asymptotic value. The results derived in this study can be useful to remote sensing applications involving IR window bands under cirrus cloud conditions.

## 1. Introduction

Cirrus clouds form a unique component of the atmosphere and significantly regulate the Earth's radiation energy budget [1-3] because of their large spatial coverage and temporal persistence. And yet, the radiative forcing of these clouds is far from being well understood because of the wide range of observed cirrus cloud properties such as height, optical thickness, particle size, and habit. With the recognition of the importance of cirrus clouds, a number of programs have been established to better investigate global cirrus properties. For instance, intensive field observations regarding cirrus clouds have been conducted as a major component of the First ISCCP Regional Experiment (FIRE) (Phase-I in October 1986 and in Phase-II in November-December 1991) [4], the European experiments on cirrus (ICE/EUCREX) carried out in 1989 [5]. Additional cirrus field campaigns are being planned, such as the Cirrus Regional Study of Tropical Anvils and Cirrus Layers (CRYSTAL). Besides these field campaigns, new instrumentation has been developed for use in space- or aircraft-borne measurement programs. Such programs include the Lidar In-space Technology Experiment (LITE) [6], the MODerate resolution Imaging Spectrometer (MODIS) [7], and the future Pathfinder Instruments for Cloud and Aerosol Spaceborne Observations-Climatologie Etendue des Nuages et des Aerosols (PICASSO-CENA) [8]. The data obtained from these instruments will enhance our capability of the detection and retrieval of cirrus clouds on a global scale with a substantial temporal span.

A number of studies have proposed methods to infer global cirrus properties from infrared (IR) radiance measurements. One benefit to using IR methods to infer cloud properties rather than those that incorporate visible or near-infrared bands is that cloud

properties are more consistent between daytime and nighttime conditions. The IR techniques are also independent of the sun glint associated with the specular reflection over water which is prevalent in daytime data. The IR CO<sub>2</sub> slicing technique is effective in inferring cirrus height and effective cloud amount (emittance multiplied by cloud fraction) [9,10]. Various IR algorithms to infer cirrus particle size have been suggested (see, e.g., Ref.[11] and [12]), whereas the methods to infer cloud thermodynamic phase using data from the 8.52, 11, and 12  $\mu\text{m}$  bands have been demonstrated by Strabala et al. [13] and Baum et al. [14]. There have been advances in recent years in the development and use of well-calibrated interferometers, such as the High Spectral Resolution Infrared Spectrometer (HIS) [15-17]. In coming years, the spectral resolution of IR data measured from space will increase greatly when interferometers are launched, such as Infrared Atmospheric Sounder Interferometer (IASI) on EUMETSAT and the Geostationary Imaging Fourier Transform Spectrometer (GIFTS) and the NPOESS (the National Polar orbiting Operational Environmental Satellite System) Aircraft Sounding Testbed-Interferometer (NAST-I) [18]. With an interferometer, cloud measurements may be recorded at thousands of wavenumbers simultaneously. To date, however, few studies have explored the use of infrared interferometer data for cirrus cloud property retrieval. One difficulty in the retrieval of cirrus cloud properties from passive airborne- or satellite-based radiometric data arises because of the difficulty in determining accurately the fundamental scattering and absorption cross sections for ice crystals over a realistic range of crystal sizes and shapes at IR bands where the applicability of the ray-tracing method breaks down [19].

Cirrus clouds are composed of almost exclusively nonspherical ice crystals, as is evident from the observations based on aircraft-borne two dimensional optical cloud probes (2D-C) and balloon-borne replicator images (see, e.g., Ref.[20] and [21]). It has been shown that the spherical approximation for nonspherical ice crystals in terms of equivalent volume or projected-area is inadequate and often misleading, as is illustrated by Liou et al. [22]. In practice, an incorrect specification of the ice crystal model in retrieving the optical thickness of cirrus clouds from satellite-borne measurements can lead to an underestimation or overestimation of the actual optical thickness by a factor that can exceed 3 [23]. Thus, nonsphericity of ice crystals must be accounted for in the development of a reliable retrieval algorithm. On this specific issue, the significance of using reliable single-scattering properties of ice crystals to generate look-up tables for retrieval implementation and the parameterization of the bulk radiative properties of cirrus clouds has been demonstrated and articulated in a number of recent publications (e.g., Ref.[14, 24-31] ). Although substantial advancements have been made in the fundamental study regarding scattering and absorption by ice crystals, as recently reviewed by Liou and Takano [24], Mishchenko et al. [34], and references cited therein, there is no a single method that can cover the entire size parameter spectrum for light scattering computations. To derive the scattering and absorption properties of nonspherical ice crystals, Liou et al. [22] developed the concept of a unified theory. This unified theory is based on a combination of a numerically accurate finite-difference time domain technique (FDTD) for small particles [35-40] and an improved geometric optics method (IGOM) [41-42] for large particles at visible and near-infrared wavelengths. However, for IR wavelengths where strong absorption is involved, there is a discontinuity

between the FDTD and IGOM results at size parameters on the order of 20, which is in practice the computational limit for the FDTD method given current computer resources. One cause of this discontinuity is the tunneling or the above-edge effect [43]. Several approximate methods [29,30, 44, 45] suggested to account for the tunneling effect are based on parameterizations involving Mie theory, a combination of Mie theory and the geometric optics method, or the complex angular momentum theory developed by Nussenzveig and Wiscombe [46,47].

The intent of this study is to develop methods capable of deriving the fundamental scattering and absorption cross sections for nonspherical ice crystals spanning a size range of 1  $\mu\text{m}$  to 10000  $\mu\text{m}$  at IR wavelengths ranging from 8  $\mu\text{m}$  to 13 $\mu\text{m}$ , where absorption effects can be appreciable. In section 2, we present methodology for calculating the scattering properties at infrared wavelengths. In particular, we present the stretched scattering potential method (SSPM) to calculate the extinction and absorption cross sections. In Section 3, we present the bulk radiative properties and parameterization for cirrus clouds. Finally, the conclusions of this study are given in Section 4.

## **2. Computation of Optical Properties for Ice Crystals at Infrared Wavelengths**

### *2.a. Aspect Ratio for Ice Crystals and Selection of Wavelengths*

A variety of nonspherical ice crystal habits, or shapes, have been observed in cirrus clouds, including hexagonal columns and plates, bullet rosettes, and complex aggregates. To demonstrate our methodology, we simplify the complexity of determining what best represents the habit distribution in cirrus by assuming cirrus are composed solely of hexagonal shapes with a random orientation in space. Studies of *in-situ* cirrus measurements have demonstrated that the upper layers of midlatitude cirrus cloud

systems are often comprised of pristine hexagonal crystals [48]. Hexagonal ice crystals are capable of producing  $22^\circ$  and  $46^\circ$  halos and other optical phenomena by scattering incident solar radiation at short wavelengths. The hexagonal shape model has been often assumed in previous studies concerning cirrus clouds [24-30, 49-51].

To carry out scattering calculations, we define the aspect ratio for ice crystals at various sizes as follows:

$$2a/L = \begin{cases} 1, & L \leq 40 \text{ } \mu\text{m} \\ \exp[-0.017835(L-40)], & 40 < L \leq 50 \text{ } \mu\text{m} \\ 5.916/L^{1/2}, & L > 50 \text{ } \mu\text{m}, \end{cases} \quad (1)$$

where  $a$  is the semi-width of cross section and  $L$  is the length of an ice crystal. The aspect ratio defined by Eq.(1) roughly corresponds to the observations reported by Ono [52] and Auer and Veal [53]. Figure 1 shows a comparison of the aspect ratio defined in this study with that of Takano and Liou [49], who used only five size-bins in discretizing the size distribution for ice crystals. Excellent agreement is evident for the comparison. In the  $D$ - $L$  plot of Fig.1 in which  $D=2a$  is the width of cross section, the continuous aspect ratio is not smooth at a size of  $L = 40 \text{ } \mu\text{m}$  because of an abrupt variation in the derivative of aspect ratio. However, this effect is very small.

To economize the computational effort, the scattering calculations are performed for 12 wavelengths between  $8 \text{ } \mu\text{m}$  and  $13 \text{ } \mu\text{m}$ , which are selected at 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 10.8, 11.0, 11.5, 12.0, 12.5, and  $13.0 \text{ } \mu\text{m}$ . The choice of these wavelengths is based on the characteristics of ice refractive index as shown in Figure 2 [54]. The dotted vertical lines indicate the locations of the selected wavelengths. Note that the variation of the refractive index is essentially linear in the intervals of the selected wavelengths. Because the refractive index is essentially linear in a given interval, the scattering and



absorption properties at any arbitrary wavelength in the 8-13  $\mu\text{m}$  region can be approximated by an interpolation of those calculated at the selected wavelengths. We also note that near 10.8  $\mu\text{m}$  there exists a region of extremely strong absorption, often called the Christiansen band [55,56]. Near 10.8  $\mu\text{m}$ , the real part of the refractive index has a value close to 1 and the extinction efficiency reaches a minimum. As is evident from Fig.2, the imaginary part of the refractive index is significant at the Christiansen band, leading to the dominance of the absorption effect in the extinction process. The behavior of ice absorption in the Christiansen band has been shown in laboratory measurements [56,57] and a theoretical explanation was reported by Yang et al. [58].

#### *2.b. Asymmetry parameter for angular distribution of scattered energy*

The angular distribution of the energy scattered by a particle is defined by its scattering phase function. At IR wavelengths, the phase function of an ice crystal is essentially featureless when compared to that at a visible or near-infrared wavelength. The primary scattering features of a hexagonal crystal at a visible wavelength, such as the halo peaks and backscattering enhancement, are smoothed out at IR wavelengths because of absorption.

For radiative transfer calculations at IR wavelengths, the detailed information of the phase function is unnecessary because the multiple scattering effect is only on the order of a few percent. Thus, the asymmetry parameter that describes the magnitude of the deviation of particle phase function from isotropic scattering is very useful at IR wavelengths, which is defined as follows:

$$g = \frac{1}{2} \int_0^\pi P(\theta) \cos\theta \sin\theta d\theta, \quad (2)$$

where  $P(\theta)$  is the normalized phase function. Given the asymmetry factor associated with an IR phase function of nonspherical ice crystals, an accurate phase function for a given size distribution of ice crystals can be approximated by the Henyey-Greenstein analytical function:

$$P_{HG}(\theta) = \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}} = \sum_{l=0}^N (2l+1) g^l P_l(\cos \theta), \quad (3)$$

where  $P_l(\cos \theta)$  is the set of Legendre polynomials derived from decomposition of the phase function. The advantage of using the Henyey-Greenstein phase function in radiative transfer calculations lies in its simplicity and efficiency in the expansion of a phase function in terms of Legendre function. There is a physical justification for such a simplification. For an IR wavelength, the phase function calculated for a given size distribution of ice crystals tends to be smooth at sidescattering and backscattering angles whereas a strong diffraction peak is typically noted in the forward scattering direction. The forward peak may need to be truncated; for this the delta-M method developed by Wiscombe [59] may be employed. The truncated phase function can be well approximated by the analytical Henyey-Greenstein function at the IR wavelengths relevant to our study.

The FDTD technique is employed to solve for the scattering and absorption properties for ice crystals whose maximum dimensions are smaller than 40  $\mu\text{m}$ . The FDTD method solves the interaction of electromagnetic waves with a particle of any given shape and is based on the difference analog of time-dependent Maxwell equations.

The strength of this method is its simplicity in concept, computational flexibility, and robust nature in dealing with light scattering problems involving nonspherical and/or inhomogeneous particles. The disadvantage of this method is its tremendous demand on computational resources when the size parameter is larger than 20. Detailed descriptions of the FDTD method may be found in Yang and Liou [38].

To calculate the asymmetry parameter for ice crystals larger than 40  $\mu\text{m}$ , we use a hybrid method based on the improved geometric optics principle (IGOM) and the electromagnetic integral equations [41, 42]. At IR wavelengths, the refractive wavelength inside an ice crystal is inhomogeneous because the planes of constant phase are not parallel to the planes of constant amplitude [60]. This inhomogeneity will affect both the ray direction and the reflection/refraction components at the air-particle interface [35]. In particular, the Fresnel coefficients are not unique when absorption is involved, as was shown recently by Yang et al. [61], who further determined the proper form of the refraction/refraction coefficients that should be used in ray-tracing calculations. We have accounted for this improvement in the present set of scattering calculations.

Figure 3 shows the asymmetry parameter calculated by the FDTD and the IGOM over a range of ice crystal sizes at wavelengths 8.5, 11, and 12  $\mu\text{m}$ . The radiative information at these three wavelengths are being used to retrieve cloud thermodynamic phase in MODIS data. For comparison of results between spherical and nonspherical particles, results computed from Mie theory for equivalent ice spheres are provided in Fig.3. Following work by Mitchell and Arnott [62], Fu et al. [29], Yang et al.[63], and Grenfell and Warren [64], we define the radius of the equivalent sphere for an individual hexagonal ice crystal as follows:

$$r_e = \frac{3}{4} \frac{\text{volume}}{\text{projected-area}} = \frac{\sqrt{3}a^2L}{\sqrt{3}a^2/2 + aL}, \quad (4)$$

where  $a$  and  $L$  the semi-width and the length of ice crystal, respectively. The advantage of the preceding definition over those for equivalent volume (or projected-area) is that both volume and area are conserved in approximating the nonspherical particle in terms of a sphere. The inserted sub-diagrams in Fig.3 provide enlargements for ice crystal sizes of 1-40  $\mu\text{m}$ . For small ice crystals with sizes of 3-9  $\mu\text{m}$ , the asymmetry parameters computed from Mie theory for the equivalent spheres and the FDTD technique for the nonspherical particles are quite different. For sizes larger than 100  $\mu\text{m}$ , the IGOM results essentially converge to the Mie results. This convergence occurs because the rays refracted into the particles are almost entirely absorbed and the diffraction and external reflection contributions dominate the scattered energy. Under random orientation conditions, diffraction and external reflection are not sensitive to the details of particle shape.

From Fig.3, we note that the FDTD results and IGOM solutions converge at a particle size of 40  $\mu\text{m}$ . Although the absolute amount of scatted and absorbed energy calculated by IGOM can have a substantial error for moderate particle sizes on the order of 40  $\mu\text{m}$  or less, the IGOM can be used to predict the relative angular distribution of scattered energy. Thus, the normalized IGOM phase function can be a good approximation. From this physical rationale and the numerical results shown in Fig.3, the concept of the unified theory developed by Liou et al. [22] can also be applied to the computation of the asymmetry parameter. Therefore, we combine the FDTD solution (for sizes smaller than 40  $\mu\text{m}$ ) and IGOM results (for sizes larger than 40  $\mu\text{m}$ ) for the asymmetry factor so that the entire size spectrum can be covered.

Figure 4 shows merged FDTD/IGOM results for the asymmetry factor for the 12 wavelengths for particle sizes of 1-10000  $\mu\text{m}$ . The variation of the asymmetry factor with the particle size displays a minimum at 30  $\mu\text{m}$  for wavelengths 8.0, 8.5 and 9.0  $\mu\text{m}$  where the absorption of ice is moderate. This minimum vanishes for longer wavelengths. The overall feature of the asymmetry parameter is that the  $g$  values are close to unity for large sizes because diffraction dominates the scattered energy which is concentrated in forward direction. For all 12 wavelengths, there is a substantial increase in the value of the asymmetry parameter as the size of the particle increases from 1  $\mu\text{m}$  to 40  $\mu\text{m}$ . For sizes larger than 40  $\mu\text{m}$ , the asymmetry parameter essentially reaches an asymptotic value.

### *2.c. Stretched Scattering Potential Method for Computing Extinction and Absorption Cross Sections for Nonspherical Particles*

Several methods exist to calculate extinction and absorption cross sections for nonspherical particles. To date, the upper limit of the size parameter region for which one can obtain the exact solution for the scattering properties of certain nonspherical particles is on the order of 200. This limitation is for axisymmetrical particles such as spheroids, finite circular cylinders, and so-called Chebyshev particles [65], as solved by T-matrix method [66]. The applicable size parameter regimes for other exact methods to solve the scattering properties of nonspherical particles are normally smaller than that associated with the T-matrix method. For a non-axisymmetrical particle, the size parameter region for which an exact solution may be obtained is substantially reduced. For instance, the limitation on size parameter for the T-matrix method is on the order of 40 when it is applied to hexagonal particles [A. Baran, personal communication]. The FDTD and its

counterpart, the discrete dipole approximation (DDA) [67,68], encounter computational difficulties for size parameters larger than 20. For these reasons, use of an approximate method is necessary in practice. The anomalous diffraction theory (ADT) developed by van de Hulst [69] has been widely used to calculate the extinction and absorption cross sections for nonspherical particles [70-72]. Unfortunately, ADT leads to substantial errors if the refractive index of the particles is not close to 1. In particular, ADT fails to account for the tunneling effect; the absorption efficiency calculated from ADT cannot be larger than 1. The intent of this section is to derive an improved approximate method to calculate the extinction and absorption cross sections.

The wave equation derived from the Maxwell equations can be written as

$$(\nabla^2 + k^2)\bar{E}(\bar{r}) = -U(\bar{r})\bar{E}(\bar{r}), \quad (5)$$

where  $U(\bar{r}) = [m^2(\bar{r}) - 1]k^2$  in which  $m$  is the complex refractive index of the medium and  $k$  is the wavenumber in a vacuum. Through comparison of Eq.(5) to the standard Schrödinger equation in quantum mechanics, we can regard  $U(\bar{r})$  as a scattering potential. Thus, the light scattering problem can be treated as one concerning the interaction of photons and a potential. A separation of variables method can be used to solve Eq.(5) exactly but is limited to only a few particle shapes such as spheres and spheroids. In practice, a high energy approximation (HEA) [73-75] or ADT can be used to solve Eq.(5) approximately. These methods are the eikonal type [76] in which rectilinear projectiles are assumed for the propagation of photons, with (or without) deviation only at the places where the scattering potential vanishes, i.e., at the particle surface. Thus, in these conventional approximations, the nonzero-interaction region where wave function undergoes phase delay and absorption is limited inside the particles.

Recently, Fu et al. [30] have computed the Poynting vector for the near-field associated with the scattering of IR radiation by a hexagonal ice crystal to illustrate the flow of electromagnetic energy around the particle. It is shown that the incident Poynting stream outside the geometric projected area of the particle can deviate and be traced through the particles, a phenomenon otherwise known as the tunneling or the above-edge effect. Although HEA and ADT provide a good physical insight in concept and a simple mathematical formulation in practice, they may produce significant errors in computation, e.g., a substantial underestimation of absorption efficiency in the resonance region because of the failure to account for the tunneling effect.

To avoid this shortcoming of the conventional eikonal-type approximation that is applied to solve Eq.(5), it is necessary to stretch the scattering potential to account for the tunneling effect. Figure 5(a) illustrates the region of non-null scattering potential, where the interaction of photons and medium occurs for the conventional eikonal methods. The scattering potential  $U(\vec{r})$  is nonzero only inside the particles for the conventional methods. Figure 5(b) is the conceptual diagram for the present stretched scattering potential method (SSPM), where  $a$  is the physical size of the particle and  $\Delta a$  is extension for the stretched potential. In this method, the scattering potential  $U(\vec{r})$  is stretched so that the non-zero effect region for photon propagation extends outside the physical volume of the scattering particle. We assume the scattering potential has a quadratic distribution, for example, in the spherical particle case, as follows:

$$U(r) = \begin{cases} (m^2 - 1)k^2, & r \leq a \\ (m^2 - 1)k^2[1 - (r - a)/\Delta a]^2, & a < r \leq a + \Delta a \\ 0, & r \leq a + \Delta a \end{cases} \quad (6)$$

For stretched scattering potential in the region between  $a$  and  $a+\Delta a$ , we can define an effective refractive index  $\tilde{m} = (\tilde{m}_r + i\tilde{m}_i)$  given by

$$\tilde{m}^2 - 1 = (m^2 - 1)k^2[1 - (r - a)/\Delta a]^2. \quad (7)$$

The solution of Eq.(7) is

$$\alpha = 1 + (m_r^2 - m_i^2 - 1)[1 - (r - a)/\Delta a]^2, \quad (8a)$$

$$\beta = m_r m_i [1 - (r - a)/\Delta a]^2. \quad (8b)$$

$$\tilde{m}_r(r) = [\alpha + (\alpha^2 + 4\beta^2)^{1/2}] / 2, \text{ and} \quad (8c)$$

$$\tilde{m}_i(r) = \beta / \tilde{m}_r(r), \quad (8d)$$

The external reflection and multiple internal reflections and subsequent transmission of the wave is completely neglected in the conventional eikonal-type approximate methods to solve Eq.(5). The result of ignoring these effects is equivalent to an overestimation of scattering potential. To include the external and internal reflection/transmission effect equivalently, the scattering potential should be compressed instead of being stretched. The compressed scattering potential is given in the form of

$$U(r) = \begin{cases} (m^2 - 1)k^2, & r \leq a - \Delta a \\ (m^2 - 1)k^2[1 - (r - a + \Delta a)/\Delta a]^2, & a - \Delta a < r \leq a. \\ 0, & r \leq a \end{cases} \quad (9)$$

Figure 6 shows the concept of using an eikonal approximation in the ADT framework to calculate the extinction and absorption cross sections for a spherical particle. In practice, continuous variation of the effective refractive index is discretized as various layers. That is, the effect of stretching the scattering potential is equivalent to adding some dielectric medium layers outside the scattering particle. For a given layer, we calculate the mean effective refractive index as follows:



$$\langle \tilde{m}_r + i\tilde{m}_i \rangle_j = \frac{1}{\Delta r} \int_{r_j - \Delta r/2}^{r_j + \Delta r/2} [\tilde{m}_r(r) + i\tilde{m}_i(r)] dr, \quad (10)$$

where  $r_j$  is the radius of the  $j$ th layer and  $\Delta r$  is the thickness of the layer. A uniform thickness is assumed for all the layers in the present study. According to the geometry shown in Fig.6 and the physical assumption in conjunction with Huygen's principle for the ADT approximation [69], we obtain expressions for the extinction cross section ( $\sigma_{ext}$ ) and the absorption cross section ( $\sigma_{abs}$ ) for a layered sphere, respectively, as follows:

$$\sigma_{ext} = 2 \iint_P (1 - e^{-\gamma} \cos \rho) d^2 P, \quad (11a)$$

$$\sigma_{abs} = \iint_P (1 - e^{-2\gamma}) d^2 P, \quad (11b)$$

where  $\rho$  is the phase delay that a photon undergoes in conjunction with its penetration of the multilayered sphere. The penetration parameter is given by

$$\rho = kd_0(m_r - 1) + \sum_{j=1}^N k(d_j - d_{j-1})(\langle \tilde{m}_r \rangle_j - 1), \quad (11c)$$

in which  $N$  is the number of total layers outside the particle. The attenuation factor,  $\gamma$ , for the damping of the incident wave is given by

$$\gamma = kd_0 m_i + \sum_{j=1}^N k(d_j - d_{j-1}) \langle \tilde{m}_i \rangle_j. \quad (11d)$$

Eqs.(11c) and (11d) are for the case where the scattering potential is stretched outside the particle. A similar mathematical formulation may be derived for the case when the scattering potential is compressed or reduced.

From the preceding discussion of the SSPM approximation, the only tuning parameter is the distance  $\Delta a$  for stretching the scattering potential. The physical processes associated with the extinction and absorption of the incident wave are quite different. For example, the surface wave (a term in the rigorous physical picture) [43, 69] that creeps along the particle surface can contribute to the extinction but not to the absorption. Therefore, the magnitude of the extension of the scattering potential may be different for calculating the extinction cross section than for the absorption cross section. For this reason, we denote  $\Delta a$  as  $\Delta a_e$  and  $\Delta a_a$  for extinction and absorption calculations, respectively.

Figure 7 shows the values of  $\Delta a_e$  that are derived from the best fit of the exact Mie solution using the SSPM results in the spherical case. In this study, the computational Mie code developed by Wiscombe [77] is used. To determine  $\Delta a_e$ , a Mie calculation is first carried out; subsequently a Monte Carlo method is employed to determine the proper value for  $\Delta a_e$  for the SSPM method. Specifically, for each SSPM calculation in the case where the potential is stretched rather than compressed, we chose

$$\Delta a_e = \xi a, \quad (12)$$

where  $\xi$  is a random number that is uniformly distributed in  $[0,1]$ . From Fig.7, it is shown that the magnitude of  $\Delta a_e$  increases with particle dimension. However, the ratio of  $\Delta a_e$  to the particle dimension reaches a maximum at a particle size of approximately  $10 \mu\text{m}$  where the resonance effect is largest. As is evident in Fig.7, the SSPM extinction efficiency actually overlaps with the Mie solution. If the procedure for calculating  $\Delta a_e$  and the extinction efficiency were reversible, the SSPM could reproduce the exact theory, providing that adequate extension of the scattering potential was known *a priori*. It is

straightforward to obtain the correct stretching scattering potential for the SSPM calculation for a spherical particle case because of the availability of the exact Mie theory, as is evident from the results shown in Fig.7. For scattering by a nonspherical particle, it is necessary to define an approximation of the correct expansion of the scattering potential.

Figure 8 is similar to Fig. 7 except it relates to the absorption calculation. Again, if  $\Delta a_a$  is properly selected, the SSPM result can match the analytical Mie solution. Another feature of the behavior of  $\Delta a_a$  as a function of the particle size is that negative values are noted for very large particles. This means that the scattering potential should be compressed in the calculation of absorption cross section because the effect of external reflection and refraction is not accounted for in the eikonal type approximation given by Eqs.(11a)-(11d). For the absorption efficiency at 12.0  $\mu\text{m}$  wavelength, a pronounced tunneling effect can be observed because the extinction efficiency is substantially larger than unity for particle sizes near 10  $\mu\text{m}$ . The SSPM accounts for the tunneling effect, providing a proper expansion of the scattering potential is used.

It is problematic to accurately predescribe the expansion of the scattering potential for SSPM calculation for a nonspherical particle. In this case, we approximate  $\Delta a_e$  and  $\Delta a_a$  by the values obtained from the equivalent spheres with radii defined by Eq.(4) that conserves both the volume and projected-area in the equivalence process. In the SSPM computation for hexagonal ice crystals, we specifically solve for the phase delay and wave attenuation for a number of layered hexagons. The total number of the hexagonal layers and their thicknesses are approximated by their counterparts in the spherical case. Figure 9 shows the absorption efficiency of hexagonal ice crystals

calculated from various methods. The errors of the FDTD results are less than 1% for the typical grid resolution [40]. Thus, the FDTD results here can be used as a reference for checking the accuracy of the approximate methods. As shown in Fig. 9, the SSPM results essentially overlap with the FDTD solution for wavelengths of 8.0, 8.5, 9.0, 9.5, 10.0, and 10.5  $\mu\text{m}$ . For longer wavelengths, deviation of the SSPM results from FDTD is noted for particle sizes larger than 20  $\mu\text{m}$ . It also becomes evident that the GOM, ADT, and MTFES may lead to substantial errors. In particular, ADT will significantly overestimate the absorption efficiency for very large particles when the particles are strongly absorptive at a given wavelength. The asymptotic value for ADT absorption efficiency is unity because the external reflection is unaccounted for in this approximation. The Mie results shown in Fig. 9 are generated for spheres whose radii are defined by Eq.(4).

Figure 10 is similar to Fig.9 except that results are shown for extinction efficiency. One obvious result is that Mie theory for the equivalent spheres substantially overestimates the extinction at the resonance maximum. The GOM and ADT methods converge for large particles. The GOM results shown here are from the improved geometric optics method developed by Yang and Liou [41, 42]. If the conventional ray-tracing method is used, the extinction efficiency is simply 2 regardless of particle size, as is pointed out by Yang and Liou [35]. For all 12 wavelengths the SSPM results seem to provide the most consistent comparisons with the FDTD method.

Some physical processes, such as the external reflection and detailed nonsphericity effects, are not fully accounted for in the SSPM because exact expansion of the potential is not used. The effects associated with these physical processes are reflected in the

equivalent Mie and GOM results to some extent. Thus, we can refine the SSPM results as follows:

$$Q_{refined} = (1 - \delta_1 - \delta_2)Q_{SSPM} + \delta_1 Q_{Mie} + \delta_2 Q_{GOM}, \quad (13)$$

where  $Q$  stands for either extinction efficiency  $Q_e$  or absorption efficiency  $Q_a$ . The values of the parameters  $\delta_1$  and  $\delta_2$  are so determined that the  $\sigma_{refined}$  best fit the FDTD results for crystal sizes between 20  $\mu\text{m}$  and 40  $\mu\text{m}$ . The refined SSPM results are given only for sizes larger than 20  $\mu\text{m}$ , because the geometric optics solution involved in Eq.(13) is essentially meaningless due to the failure of the localization principle for small size parameters. Figure 11 shows the refined SSPM results for the extinction and absorption cross section efficiencies. With this refinement, calculations can be provided for the entire size spectrum. The refinement procedure employed here is similar to that used by Fu et al. [29] who use a weighted summation of Mie and GOM solutions to obtain an approximate fit for the single-scattering properties of hexagonal ice crystals.

### 3. Application to Cirrus Clouds and Parameterizations

To derive a set of ice crystal bulk scattering properties for practical applications, it is necessary to derive the single-scattering properties using realistic particle size distributions. In this study, we select the 28 size distributions used by Fu [28] with an additional two datasets from Mitchell et al.[78]. These size distributions were measured for a variety of midlatitude and tropical cirrus clouds and were obtained during various field campaigns including the Central Equatorial Pacific Experiment (CEPEX) [79]. McFarquhar et al. [80] showed that the microphysical properties of ice crystals for tropical cirrus clouds are quite different from those for midlatitude cirrus systems.

It is common to characterize the bulk properties of a size distribution by two parameters – ice water content (*IWC*) and effective size. For a given size distribution, *IWC* is defined as

$$IWC = \rho_{ice} \int_{L_{min}}^{L_{max}} V(L)n(L)dL, \quad (14)$$

where  $\rho_{ice}$  is the mass density of bulk ice,  $n(L)$  is particle number density,  $V(L)$  is the volume of an ice crystal with maximum dimension of  $L$ , and  $L_{min}$  and  $L_{max}$  are the minimum and maximum sizes in the size distributions, respectively. There are many definitions for effective size in the literature. Wyser and Yang [31] performed a comprehensive comparison of definitions commonly used in different parameterization efforts. In this study, we define the effective size following Foot [81], Francis et al. [5], Fu [28], Wyser and Yang [31], and Grenfell and Warren [64] as

$$D_e = \frac{3 \int_{L_{min}}^{L_{max}} V(L)n(L)dL}{2 \int_{L_{min}}^{L_{max}} A(L)n(L)dL}, \quad (15)$$

where  $A(L)$  is the projected-area of the particle with size of  $L$ . The preceding definition reduces to that defined by Hansen and Travis [82] for spherical particles. We note that Eq.(15) is similar to Eq.(4) except that only an individual particle is considered in Eq.(4) whereas Eq.(15) considers a size distribution. In addition to the effective size, we also define the mean maximum dimension for a given size distribution as follows:

$$\langle L \rangle = \frac{\int_{L_{min}}^{L_{max}} Ln(L)dL}{\int_{L_{min}}^{L_{max}} n(L)dL}. \quad (16)$$

Figure 12 shows a scatterplot for the effective size  $D_e$  as a function of the mean maximum dimension  $\langle L \rangle$  for the aforementioned thirty size distributions described. It is evident that  $D_e$  is normally larger than  $\langle L \rangle$  for a given size distribution. This occurs because the distributions contain a large number of small particles that are significant in defining the mean maximum dimension. However, the contribution of small particles to the total volume or projected-area is relatively small regardless of their contribution. Thus, the largest weight in defining  $D_e$  is for moderate or large ice crystals. It is also evident from Fig. 12 that the spectrum of effective sizes ranges from approximately 10  $\mu\text{m}$  to 160  $\mu\text{m}$  for the selected size distribution datasets. Given the range of effective sizes, the present set of ice cloud size distributions should provide an effective basis for a parameterization effort.

For a given size distribution of ice crystals, the bulk extinction coefficient is defined as follows:

$$\beta_e = \int_{L_{\min}}^{L_{\max}} Q_e(L) A(L) n(L) dL. \quad (17)$$

It is convenient in practice to define a volume-normalized extinction coefficient [31], given by

$$\tilde{\beta}_e = \frac{\beta_e}{(IWC/\rho_{ice})} = \frac{\int_{L_{\min}}^{L_{\max}} Q_e(L) A(L) n(L) dL}{\int_{L_{\min}}^{L_{\max}} V(L) n(L) dL}. \quad (18)$$

Note that the quantity  $(IWC/\rho_{ice})$  in the preceding equation provides the total bulk volume of ice crystals for a given size distribution. For a cirrus cloud with a given  $IWC$

and geometrical thickness  $\Delta z$ , the dimensionless optical thickness of the cloud may be obtained from

$$\tau = \tilde{\beta}_e \frac{IWC}{\rho_{ice}} \Delta z. \quad (19)$$

This relationship is useful in practice because  $IWC$  is now a prognostic parameter in many global climate models (GCMs) [83]. In many parameterization efforts, it is common to parameterize  $\beta_e$  or  $\tilde{\beta}_e$  as a function of the effective size. In particular, it has been found that the extinction coefficient decreases with an increase in effective size (e.g., Ref.[28,31] ). We suggest that it may be more useful to parameterize the mean extinction efficiency rather than the extinction coefficient. The volume-normalized extinction coefficient  $\tilde{\beta}_e$  can be rewritten exactly as

$$\tilde{\beta}_e = \frac{3}{2D_e} \langle Q_e \rangle, \quad (20)$$

where  $\langle Q_e \rangle$  is the mean extinction efficiency that is defined as

$$\langle Q_e \rangle = \frac{\int_{L_{min}}^{L_{max}} Q_e(L) A(L) n(L) dL}{\int_{L_{min}}^{L_{max}} A(L) n(L) dL}. \quad (21)$$

From inspection of Eqs. (20) and (21), we suggest that the accuracy of a parameterization scheme would be improved if the mean extinction efficiency could be parameterized accurately.

For visible wavelengths, the size parameter is very large even for small ice crystals on the order of 10  $\mu\text{m}$ . Thus, for visible wavelengths, the extinction efficiency of an ice crystal essentially approaches a geometric optics asymptotic value of 2 for a realistic set of particle sizes. For the IR region of interest in the present study, the resonance effect is



obvious for ice crystal sizes between 10  $\mu\text{m}$  and 20  $\mu\text{m}$ . However, the number concentration of particles in this size region may be significant for some ice clouds such as contrails and cold cirrus. Thus,  $\langle Q_e \rangle$  needs to be investigated carefully in a parameterization effort. From Eq.(20), the wavelength-dependent part of the extinction coefficient is the mean extinction efficiency. The mean extinction efficiency can be parameterized as a function of effective size.

Figure 13 shows the variation of the volume-normalized extinction coefficient,  $\tilde{\beta}_e$ , as a function of effective size for 12 wavelengths in the IR window region. The extinction coefficient decreases with an increase of effective particle size, as expected from Eq.(20). For effective particle sizes larger than 80  $\mu\text{m}$ , the extinction coefficient approaches its asymptotic value with a small variation as particle size increases. We note that  $\tilde{\beta}_e$  can be regarded as the extinction coefficient for a unity *IWC* because the mass density of ice is independent of the size distribution. For a fixed cloud geometrical thickness and *IWC*, according to Fig.13 and Eq.(19), the cloud will have a larger optical thickness if the ice crystals are small. Conversely, the optical thickness of the cloud will be reduced if the particle sizes are large.

The circle symbols in Fig.14 show the variation of the dimensionless mean extinction efficiency as a function of  $1/D_e$ . The variation pattern of  $\langle Q_e \rangle$  is shown to have a dependence on wavelength. The magnitude of  $\langle Q_e \rangle$  ranges between 1.5 and 2.2. When  $1/D_e$  approaches zero (*i.e.*, large effective size), the mean effective extinction efficiency is close to 2. This is expected because the particle extinction efficiency reaches its geometric optics asymptotic value when the size is very large. For wavelengths 8.5, 9.0, 9.5, 10.0, 12.0, 12.5 and 13.0  $\mu\text{m}$ , an extinction maximum is

observed for the moderate effective sizes. For wavelengths 10.5, 10.8, 11.0, and 11.5  $\mu\text{m}$  near the Christiansen band, the mean extinction efficiency decreases with the increase of  $1/D_e$ , i.e., a decrease of particle mean size.

The mean extinction efficiency may be parameterized by a second-order polynomial in the form of

$$\langle Q_e \rangle = \eta_2(1/D_e)^2 + \eta_1(1/D_e) + \eta_0, \quad (22)$$

where the coefficients  $\eta_0$ ,  $\eta_1$ , and  $\eta_2$  are determined from the least-squared fitting technique. Table 1 lists the three fitting coefficients derived for the 12 wavelengths. The parameterization results are also shown in Fig. 14 (solid lines).

The circle symbols in Fig.15 show the variation of single-scattering albedo with  $1/D_e$ . Here the mean value of the single-scattering albedo for a given size distribution is defined as follows:

$$\langle \tilde{\omega} \rangle = \frac{\int_{L_{\min}}^{L_{\max}} [Q_e(L) - Q_a(L)] A(L) n(L) dL}{\int_{L_{\min}}^{L_{\max}} Q_e(L) A(L) n(L) dL}, \quad (23)$$

The overall variation trend of  $\langle \tilde{\omega} \rangle$  in the IR window region can be grouped into two categories: for wavelengths smaller than 10.0  $\mu\text{m}$ , the single-scattering albedo increases with an increase of  $1/D_e$ , i.e., single-scattering albedo increases with decreasing mean particle size; for wavelengths longer than 10.5, the opposite behavior is observed. A second-order polynomial function may be employed to fit the single-scattering albedo in the form of

$$\langle \tilde{\omega} \rangle = \zeta_2(1/D_e)^2 + \zeta_1(1/D_e) + \zeta_0. \quad (24)$$

Table 2 lists the fitting coefficients in Eq.(24). The solid lines in Fig. 15 show the corresponding parameterization results.

The circle symbols in Fig. 16 show the asymmetry factor calculated for the 30 size distributions at 12 wavelengths. In this figure, the asymmetry factor is plotted against the effective size rather than against  $1/D_e$  as in Figs. 14 and 15. The asymmetry factor increases with an increase in the effective size. One explanation for this is that particle absorption reduces the amount of scattered energy in the side scattering and back scattering directions, and affects the transmission of the incident wave. In addition, the diffracted energy is concentrated in a narrower region around the forward scattering direction when the particle sizes are increased.

While polynomial functions are employed often to parameterize the asymmetry factor (e.g.,Ref.[28,31]), we suggest instead the use of a power law form, which may be mathematically expressed as follows:

$$\langle g \rangle = \zeta (\langle D_e \rangle)^\kappa. \quad (25)$$

The fitting coefficients  $\zeta$  and  $\kappa$  for Eq.(25) are listed in Table 3. Our parameterization results as given by Eq.(22), (24) and (25) should be limited to the domain of effective sizes (approximately 10-160  $\mu\text{m}$ ) used in our analyses. For an effective size outside of this range, the applicability of the parameterization should be checked discreetly to avoid any unpleasant artifacts.

## Conclusions

We present fundamental scattering and absorption properties for hexagonal ice crystals with sizes ranging from 1  $\mu\text{m}$  to 10000  $\mu\text{m}$  in the infrared (IR) 8-13  $\mu\text{m}$  window region. The 8-13  $\mu\text{m}$  region contains a wealth of spectral information that may be

exploited by satellite-borne IR retrieval of ice cloud properties. To obtain scattering and absorption properties over this range of particle sizes, two methods need to be employed because there is currently no single scattering computational method that can cover the entire size parameter spectrum. In this study we use the finite-difference time domain method to solve for the extinction efficiency, single-scattering albedo, and the asymmetry parameter of the phase function for ice crystals smaller than  $40\text{ }\mu\text{m}$ . We find that the improved geometric optics method (IGOM) can be employed to calculate the asymmetry parameter for ice crystals larger than  $40\text{ }\mu\text{m}$  if one accounts for the inhomogeneity effect of the refracted wave inside the nonspherical ice particle. The combination of the two methods provide the results for the entire range of particle sizes over wavelengths ranging from  $8\text{ }\mu\text{m}$  to  $13\text{ }\mu\text{m}$ .

Other methods are compared but shortcomings are noted. In particular, the geometric optics and the anomalous diffraction theory fail to account for the tunneling effect (a phenomenon that the incident energy stream outside the particle projected-area can be trapped and scattered/absorbed) in calculating the absorption and extinction efficiency. The analytical Mie theory does account for the tunneling effect. When the equivalent spherical approximation is applied to scattering by a nonspherical ice crystal, a significant overestimation of the effect can result. This is a particular problem at the size parameter where the resonance maximum is produced. For the extinction and absorption efficiency calculations, several methods including the IGOM, the Mie solution for equivalent spheres, and the anomalous diffraction theory (ADT) can lead to a substantial discontinuity at a particle size of  $40\text{ }\mu\text{m}$ .

We have developed a novel new approach called the stretched scattering potential method (SSPM) to address the aforementioned difficulties. At 12 wavelengths in the spectral region 8-13  $\mu\text{m}$ , we show that the SSPM is a more accurate approximation than ADT, Mie theory, and IGOM. Additionally, we suggest further numerical refinements to the SSPM solution. Through a combination of the FDTD and SSPM, we have computed the extinction and absorption efficiency for hexagonal ice crystals with sizes ranging from 1  $\mu\text{m}$  to 10000  $\mu\text{m}$  at 12 wavelengths between 8-13  $\mu\text{m}$ .

Based on the single-scattering properties obtained for individual ice crystals, 30 size distributions obtained from various field campaigns for midlatitude and tropical cirrus cloud systems have been selected to calculate the bulk scattering/absorption properties for the clouds. A further parameterization effort is carried out to analytically fit these bulk scattering properties by using second-order polynomial functions for the extinction efficiency and the single-scattering albedo and a power law form for the asymmetry parameter. We note that the volume-normalized extinction coefficient can be separated into two parts: one is inversely proportional to effective size and is independent of wavelength, and the other is mean effective extinction efficiency. Unlike conventional parameterization efforts, in the present parameterization scheme the latter part of the volume-normalized extinction coefficient is numerically fitted. Finally, it should be pointed out that single-scattering properties of ice crystals in the IR window region show a very strong wavelength-dependence, in particular, in the case of single-scattering albedo. Numerical results show that the single-scattering albedo increase with the decrease of particle sizes for wavelengths shorter than 10.0  $\mu\text{m}$  and opposite pattern is

observed for longer wavelengths. The results obtained in this study can be useful for spaceborne IR retrieval of cirrus clouds.

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## References

- [1] Liou KN. Influence of cirrus clouds on weather and climate processes: A global perspective. *Mon Wea Rev* 1986; 114:1167-119.
- [2] Stephens GL, Tsay SC, Stackhose PW, Flatau PJ. The relevance of the microphysical and radiative properties of cirrus clouds to climate and climate feedback. *J Atmos Sci* 1990; 47: 1742-1753.
- [3] Randall DA, Harshvardhan, Dazlich DA, Gorsetti TG. Interactions among radiation, convection, and large-scale dynamics in a general circulation model. *J Atmos Sci* 1989; 46: 1943-1970.
- [4] Starr DO. A cirrus cloud experiment: Intensive field observations planned for FIRE. *Bull Amer Meteor Soc* 1987; 68:119-124.
- [5] Francis PN, Jones A, Saunders RW, Shine KP, Slingo A, Sun Z. An observational and theoretical study of the radiative properties of cirrus: Some results from ICE'89. *Quart J Roy Meteor Soc* 1994; 120: 809-848.
- [6] Winker DM, Couch RH, McCormick MP. An overview of LITE: NASA's Lidar In-space Technology Experiment. *Proc IEEE* 1996; 84: 164-180.
- [7] King MD, Kaufman YJ, Menzel WP, Tanre D. Remote sensing of cloud, aerosol, and water vapor properties from the Moderate Resolution Imaging Spectrometer (MODIS). *IEEE Trans. Geosci. Remote Sensing* 1992; 30: 2-26.
- [8] Winker DM, Wielicki BA. The PICASSO-CENA Mission. in *Sensors, Systems, and Next-Generation Satellites III*. *Proc. SPIE* 1999; 3870: 26-36.
- [9] Menzel

- [10] Fray
- [11] Stubenrauch CJ, Holz R, Chedin A, Mitchell DL, Baran AJ, Retrieval of cirrus ice crystal sizes from 8.3 and 11.1  $\mu\text{m}$  emissivities determined by the improved initialization inversion of TIROS-N Operational Vertical Sounder observations. J Geophys Res 1999; 104: 31793-31808.
- [12] Chung S, Ackerman SA, van Delst PF, Menzel WP. Model calculations and interferometer measurements of ice-cloud characteristics. J Appl Meteor 2000; 39: 634-644.
- [13] Strabala KI, Ackerman SA, Menzel WP. Cloud properties inferred from 8-12  $\mu\text{m}$  data. J Appl Meteor 1994; 2:212-229.
- [14] Baum B.A., Soulen PF, Strabala KI, King MD, Ackerman SA, Menzel WP, Yang P. Remote sensing of cloud properties using MODIS Airborne Simulator imagery during SUCCESS. II. Cloud thermodynamic phase. J Geophys Res 2000; 105: 11781-11792
- [15] Revercomb HE, Buijs H, Howell HB, Laporte DD, Smith WL, Sromovsky LA. Radiometric calibration of IR Fourier transform spectrometers: Solution to a problem with the High Spectral Resolution Interferometer Sounder. Appl Opt 1998;27:3210-3218.
- [16] Smith WL, Revercomb HE, Knuteson RO, Best FA, Dedecker R, Howell HB, H. Woolf M. Cirrus cloud properties derived from high spectral resolution infrared spectrometry during FIRE II. Part I: The high resolution interferometer sounder (HIS) systems. J Atmos Sci 1995; 52: 4238-4245.



- [17] Ackerman SA, Smith WL, Spinhirne JD, Revercomb HE. The 27-28 October 1986 FIRE IFO cirrus case study: Spectral properties of cirrus clouds in the 8-12  $\mu\text{m}$  window. *Mon Wea Rev* 1990; 118: 2377-2388.
- [18]
- [19] Takano Y, Liou KN, Minnis P. The effects of small ice crystals on cirrus infrared radiation properties. *J Atmos Sci* 1992; 49: 1487-1493.
- [20] Heymsfield AJ, Platt CMR. A parameterization of the particle size spectrum of ice clouds in terms of the ambient temperature and the ice water content. *J Atmos Sci* 1984; 41: 846-856.
- [21] Mitchell DL, Macke A, Liu Y. Modeling cirrus clouds, II, Treatment of radiative properties. *J Atmos Sci* 1996; 53: 2967-2988.
- [22] Liou KN, Takano Y, Yang P. Light scattering and radiative transfer in ice crystal clouds: Applications to Climate Research. In: Mishchenko MI, Hovenier JW, Travis LD, editors. *Light Scattering by Nonspherical Particles: theory, measurements, and applications*. San Diego: Academic Press, 1999.
- [23] Mishchenko MI, Rossow WB, Macke A, Lacis AA. Sensitivity of cirrus cloud albedo, bidirectional reflectance and optical thickness retrieval accuracy to ice particle shape. *J Geophys Res* 1996; 102: 16973-16985.
- [24] Liou KN, Takano Y. Light scattering by nonspherical particles: remote sensing and climatic implications. *Atmos Res* 1994; 31: 271-298.
- [25] Minnis P, Liou KN, Takano Y. Inference of cirrus cloud properties using satellite-observed visible and infrared radiances, I. Parameterization of radiance fields. *J Atmos Sci* 1993; 50: 1279-1304.

- [26] Minnis P, Liou KN, Takano Y. Inference of cirrus cloud properties using satellite-observed visible and infrared radiances, II, Verification of theoretical cirrus radiative properties. *J Atmos Sci* 1993; 50:1305-1322.
- [27] Han Q, Rossow WB, Chou J, Kuo KS, Welch RM. The effects of aspect ratio and surface roughness on satellite retrievals of ice-cloud properties. *J Quant Spectrosc Radiat Transfer* 1999;63:559-583.
- [28] Fu Q. An accurate parameterization of the solar radiative properties of cirrus clouds for climate models. *J Climate* 1996; 9:2058-2082.
- [29] Fu Q, Yang P, Sun WB. An accurate parameterization of the infrared radiative properties of cirrus clouds for climate models. *J Climate* 1998; 25: 2223-2237.
- [30] Fu Q, Sun WB, Yang P. On modeling of scattering and absorption by cirrus nonspherical ice particles at thermal infrared wavelengths. *J Atmos Sci* 1999; 56:2937-2947.
- [31] Wyser K, Yang P. Average Crystal size and Bulk shortwave single scattering properties in ice clouds. *J Atmos Res* 1998; 49:315-335.
- [32] Baum BA, Kratz D, Yang P, Ou SC, Hu YX, Soulen PF, Tsay SC. Remote sensing of cloud properties using MODIS Airborne Simulator imagery during SUCCESS. I. Data and models. *J Geophys Res* 2000; 105: 11767-11780.
- [33] Baum BA, Spinhirne J. Remote sensing of cloud properties using MODIS Airborne Simulator imagery during SUCCESS. III. Cloud overlap. *J Geophys Res* 2000; 105: 11793-11804.

- [34] Mishchenko MI, Hovenier JW, Travis LD, editors. Light Scattering by Nonspherical particles: Theory, Measurements, and Applications. San Diego: Academic Press, 1999
- [35] Yang P, Liou KN. Light scattering by hexagonal ice crystals: comparison of finite-difference time domain and geometric optics methods. J Opt Soc Amer A 1995; 12: 162-176.
- [36] Yang P, Liou KN. Finite-difference time domain method for light scattering by small ice crystals in three-dimensional space. J Opt Soc Amer A 1996; 13: 2072-2085.
- [37] Yang P, Liou KN. An efficient algorithm for truncating spatial domain in modeling light scattering by finite-difference technique. J Comput Phys 1998; 140: 346-269.
- [38] Yang P, Liou KN. Finite-difference time domain method for light scattering by nonspherical and inhomogeneous particles: theory, measurements, and applications. In: Mishchenko MI, Hovenier JW, Travis LD, editors. Light Scattering by Nonspherical Particles: theory, measurements, and applications. San Diego: Academic Press, 1999.
- [39] Sun WB, Fu Q, Chen Z. Finite-difference time-domain solution of light scattering by dielectric particles with a perfectly matched layer absorbing boundary condition. Appl Opt 1999; 28: 3141-3151.
- [40] Yang P, Liou KN, Mishchenko MI, Gao BC. Efficient finite-difference time-domain scheme for light scattering by dielectric particles: application to aerosols. Appl Opt 2000; 39: 3727-3737.

- [41] Yang P, Liou KN. Geometric-Optics-integral-equation method for light scattering by nonspherical ice crystals. *Appl Opt* 1996; 35: 6568-6584.
- [42] Yang P, Liou KN. Light scattering by hexagonal ice crystals: Solution by ray-by-ray integration algorithm. *J Opt Soc Amer A* 1997; 14: 2278-2289.
- [43] Nussenzveig HM. Diffraction effects in semiclassical scattering. New York: Cambridge University Press, 1992.
- [44] Mitchell DL. Parameterization of the Mie extinction and absorption coefficients for water clouds. *J Atmos Sci* 2000; 57: 1311-1326.
- [45] Baran AJ, Havemann S. Rapid computation of the optical properties of hexagonal columns using complex angular momentum theory. *J Quant Spectrosc Radiat Transfer* 1999;63:499-519.
- [46] Nussenzveig HM, Wiscombe WJ. Efficiency factors in Mie scattering. *Phys Rev Lett* 1980;45:1490-1494.
- [47] Nussenzveig HM, Wiscombe WJ. Diffraction as tunneling. *Phys Rev Lett* 1987;59:1667-1670.
- [48] Heymsfield AJ, Iaquinta J. Cirrus crystal terminal velocities. *J Atmos Sci* 2000; 57:916-938.
- [49] Takano Y, Liou KN. Radiative transfer in cirrus clouds. I Single-scattering and optical properties of hexagonal ice crystals. *J Atmos Sci* 1989; 46: 3-19.
- [50] Takano Y, Liou KN. Radiative transfer in cirrus clouds. II. Theory and computation of multiple scattering in an anisotropic medium. *J Atmos Sci* 1989; 46: 20-36.

- [51] Ebert EE, Curry JA. A parameterization of ice cloud optical properties for climate models. *J Geophys Res* 1992; 97: 3831-3836.
- [52] Ono A. The shape and riming properties of ice crystals in natural clouds. *J Atmos Sci* 1969; 26: 138-147.
- [53] Auer AH Jr, Veal DL. The dimension of ice crystals in natural clouds. *J Atmos Sci* 1970; 27: 919-926.
- [54] Warren S. Optical constants of ice from the ultraviolet to the microwave. *Appl Opt* 1984; 23: 1206-1225.
- [55] Carlon HR. Christiansen effect in IR spectra of soil-derived atmospheric dusts. *Appl Opt* 1976; 18: 3610-3614.
- [56] Arnott WP, Dong YY, Hallett J. Extinction efficiency in the infrared (2-18  $\mu\text{m}$ ) of laboratory ice clouds: Observations of scattering minima in the Christiansen bands of ice. *Appl Opt* 1995; 34: 541-551.
- [57] Schmitt CG, Arnott WP. Infrared emission (500-2000  $\text{cm}^{-1}$ ) of laboratory ice clouds. *JQSRT* 2000; 63: 701-725.
- [58] Yang P, Liou KN, Arnott WP, 1997: Extinction efficiency and single-scattering albedo of ice crystals in laboratory and natural cirrus clouds. *J Geophys Res* 1997; 102: 21825-21835.
- [59] Wiscombe WJ. The delta-M method: rapid yet accurate radiative flux calculations for strongly asymmetric phase functions. *J Atmos Sci* 1977; 34:1408-1422.
- [60] Stratton JA. *Electromagnetic Theory*. New York: McGraw-Hill, 1941.

- [61] Yang P, Gao BC, Baum BA, Hu YX, Wiscombe WJ, Mishchenko MI, Winker DM, Nasiri SL. Asymptotic solutions for optical properties of large particles with strong absorption. 2000; Appl Opt (submitted).
- [62] Mitchell DL, Arnott WP. A model predicting the evolution of ice particle size spectra and radiative properties of cirrus clouds. Part II: Dependence of absorption and extinction on ice crystal morphology. J Atmos Sci 1994;51:817-832.
- [63] Yang P, Liou KN, Wyser K, Mitchell D. Parameterization of the scattering and absorption properties of individual ice crystals. J Geophys. Res 2000;105:4699-4718.
- [64] Grenfell TC, Warren SG. Representation of a nonspherical ice particle by a collection of independent spheres for scattering and absorption of radiation. J Geophys Res 1999;104:31697-31709.
- [65] Mugnai A, Wiscombe WJ. Scattering from nonspherical Chebyshev particles. 1. Cross sections, single-scattering albedo, asymmetry factor, and backscattered fraction. Appl Opt 1986;25:123-1244.
- [66] Mishchenko MI. Light scattering by size-shape distributions of randomly oriented axially symmetric particles of a size comparable to a wavelength. Appl Opt 1993;32:4652-4666.
- [67] Purcell EM Pennypacker CR. Scattering and absorption of light by nonspherical dielectric grains. Astrophys J 1973;186:705-714.
- [68] Draine BT, Flatau PJ. Discrete-dipole approximation for scattering calculations. J Opt Soc Am A 1994;11:1491-1499.

- [69] van de Hulst HC. Light scattering by small particles. New York: Wiley 1957.
- [70] Chylek P, Klett JD. Extinction cross sections of nonspherical particles in the anomalous diffraction approximation. *J Opt Soc Am A* 1991;8:274-281.
- [71] Liu Y, Arnott WP, Hallett J. Anomalous diffraction theory for arbitrarily oriented finite circular cylinders and comparison with exact T-matrix results. *Appl Opt* 1998;37:5019-5030.
- [72] Sun WB, Fu Q. Anomalous diffraction theory for arbitrarily oriented hexagonal crystals. *J Quant Spectrosc Radiat Transfer* 1999; 63:727-737.
- [73] Schiff LI. Approximation method for short wavelength or high-energy scattering. *Phys Rev* 1956;104:1481-1485.
- [74] Glauber RJ. High-Energy Collision Theory. In: Britten WE, Dunham LC, editors. *Lectures in Theoretical Physics*. New York: Interscience, 1959.
- [75] Chen TW. High energy light scattering in the generalized eikonal approximation. *Appl Opt* 1989;28:4096-4102.
- [76] Perrin JM, Chiappeta P. Light scattering by large particles I. A new theoretical description in the eikonal picture. *Opt Acta* 1985;32:907-921.
- [77] Wiscombe WJ. Improved Mie scattering algorithms. *Appl Opt* 1980;19:1505-1509.
- [78] Mitchell DL, Chai SK, Liu Y, Heymsfield AJ, Dong Y. Modeling cirrus clouds. Part I: treatment of bimodal size spectra and case study analysis. *J Atmos Sci* 1996;53:2952-2966.

- [79] McFarquhar GM, Heymsfield AJ. Microphysical characteristics of three cirrus anvils sampled during the Central Equatorial Pacific Experiment. *J Atmos Sci* 1996;53:2401-2423.
- [80] McFarquhar GM, Heymsfield AJ, Macke A, Iaquinta J, and Aulenbach SM. Use of observed ice crystal sizes and shapes to calculate mean scattering properties and multi-spectral radiances: CEPEX 4 April 1993 case study. *J Geophys Res* 1999;104:31763-31779.
- [81] Foot JS. Some observations of the optical properties of clouds. Part II: Cirrus. *Quart J Roy Meteor Soc* 1988;114:145-164.
- [82] Hansen JE, Travis LD. Light scattering in planetary atmosphere. *Space Sci Rev* 1974;16:527-610.
- [83] Liou KN, Yang P, Takano Y, Sassen K, Charlock T, Arnott W. On the radiative properties of contrail cirrus. *Geophys. Res Lett* 1998;25:1161-1164.



## Figure Captions

- Fig.1 The aspect ratio used for defining the geometry of the hexagonal ice crystals.  $D$  and  $L$  are the width of cross section and length of the particle, respectively.
- Fig.2 The variation of refractive index in IR (7.5-13.5  $\mu\text{m}$ ) region. The refractive index data are those compiled by Warren [1984]. The dotted lines in the diagrams indicate the locations of the wavelengths selected for the presented scattering computation.
- Fig.3 The asymmetry parameter computed from the FDTD technique and the improved geometric optics method for hexagonal ice crystals at three wavelengths. The subdiagrams are the enlargements for particle sizes between 1 to 40  $\mu\text{m}$ . The Mie solutions are for the equivalent spheres whose radii are defined by Eq. (4).
- Fig.4 The combined FDTD and geometric optics solutions for the asymmetry factor at 12 wavelengths, which cover the particle sizes from 1 to 10000  $\mu\text{m}$ .
- Fig.5 The conceptual diagrams in spherical case to illustrate the principle of the conventional eikonal-type approximation and the present stretching scattering potential method (SSPM). (a): the non-zero effect region for phase delay and wave attenuation in the conventional method. The radius of the sphere is denoted by  $a$ . (b): the region of nonzero scattering potential in the SSPM. The scattering potential is expanded outside the particle in the region between  $a$  and  $a + \Delta a$  to account for the tunneling effect.
- Fig.6 The conceptual diagram to illustrate the computational scheme for multi-layered particle in the frame of an eikonal-type approximate method. Rectilinear

projectiles are assumed for the of photon paths in conjunction with the computation of phase delay and wave attenuation.

Fig.7 The expansion parameter  $\Delta a_e$  derived from the best fit of the SSPM solutions to the exact Mie results in spherical case at wavelengths 8.5, 11.0, and 12.0  $\mu\text{m}$ . The values of SSPM extinction efficiency corresponding to the best fitting  $\Delta a_e$  are also shown in comparison with the Mie results. The parameter  $\Delta a_e$  are presented in an absolute scale (in the unit of incident wavelength) and also the percentage with respect to the particle radii.

Fig.8 As the same as Fig.7 except for absorption efficiency. Pronounced tunneling effect is evident from the result for 12.0  $\mu\text{m}$  wavelength, as the absorption efficiency is substantially larger than unity at particle radii near 10  $\mu\text{m}$ .

Fig.9 The absorption of hexagonal ice crystals computed from various methods. The particle sizes are defined by their maximum dimensions with an implicit inclusion of aspect ratio defined in Eq. (1). The expansion parameter  $\Delta a_e$  and  $\Delta a_a$  required for SSPM calculation in the hexagonal case are approximated by their counterparts in a spherical case, that is, the number of layers of stretched potential and their thickness in hexagonal case is the same as in the spherical case.

Fig.10 As the same as Fig.9 except for extinction efficiency.

Fig.11 The combination of the FDTD and refined SSPM results for the absorption and extinction efficiency for the particle sizes ranged from 1 to 10000  $\mu\text{m}$ . A smooth transition between the two methods are evident.

- Fig.12 The effective sizes and mean maximum dimensions computed for the 30 size distributions that are taken from Fu [1996] and Mitchell et al. [19996].
- Fig.13 The variation of the volume-normalized extinction coefficient versus the effective size for 12 wavelengths in the IR window region.
- Fig.14 The mean extinction efficiency calculated for the 30 size distributions (circle symbols). The solid lines are the parameterization results obtained from fitting the “exact” data in terms of the second order polynomial function of  $1/D_e$ .
- Fig.15 As the same as Fig.14 except for mean single-scattering albedo. An increasing trend is evident for the variation of the single-scattering albedo versus  $1/D_e$  at wavelengths 8.0, 8.5, 9.0, 9.5 and 10.0  $\mu\text{m}$ . The opposite is noted for the other wavelengths.
- Fig.16 The variation of the asymmetry factor versus  $D_E$  for the 12 wavelengths (circle symbols). The solid lines are the parameterization results based on power law fitting

Table 1. The fitting coefficients for the parameterization of the mean extinction efficiency, which is defined by the Eq.(22).

Wavelength ( $\mu\text{m}$ )	$\eta_0$	$\eta_1$	$\eta_2$
8.0	1.989	4.336	-2.776E+1*
8.5	1.979	6.117	-4.877E+1
9.0	1.982	5.219	-4.248E+1
9.5	1.992	3.891	-3.743E+1
10.0	1.984	5.208	-8.194E+1
10.5	2.032	-3.643	-4.075E+1
10.8	2.036	-6.777	5.749
11.0	2.029	-5.179	3.789
11.5	2.012	-1.431	-1.061E+1
12.0	2.006	1.283	-2.800E+1
12.5	2.000	2.787	-3.323E+1
13.0	1.998	3.201	-2.664E+1

\*: 2.776E+1 indicates  $2.776 \times 10^{+1}$ .

Table 2. The fitting coefficients for the parameterization of the mean single scattering albedo, which is defined by the Eq.(24).

Wavelength ( $\mu\text{m}$ )	$\zeta_o$	$\zeta_1$	$\zeta_2$
8.0	5.294E-1	2.536E-1	1.541E+1
8.5	5.205E-1	1.421	7.036
9.0	5.198E-1	1.178	9.676
9.5	5.191E-1	1.040	1.063E+1
10.0	5.116E-1	1.770	-1.811
10.5	5.230E-1	-8.619E-1	1.271
10.8	5.384E-1	-2.454	1.225E+1
11.0	5.475E-1	-2.552	1.417E+1
11.5	5.562E-1	-1.974	9.011
12.0	5.633E-1	-1.919	9.232
12.5	5.556E-1	-1.605	7.246
13.0	5.678E-1	-2.051	1.237E+1

Table 3. The fitting coefficients for the parameterization of the mean asymmetry parameter, which is defined by the Eq.(25).

Wavelength ( $\mu\text{m}$ )	$\zeta$	$\kappa$
8.0	8.317E-1	3.279E-2
8.5	8.217E-1	3.565E-2
9.0	8.399E-1	3.120E-2
9.5	8.560E-1	2.779E-2
10.0	8.917E-1	2.092E-2
10.5	9.271E-1	1.387E-2
10.8	9.092E-1	1.771E-2
11.0	9.080E-1	1.501E-2
11.5	8.922E-1	1.394E-2
12.0	8.519E-1	2.355E-2
12.5	8.553E-1	1.932E-2
13.0	8.290E-1	2.682E-2

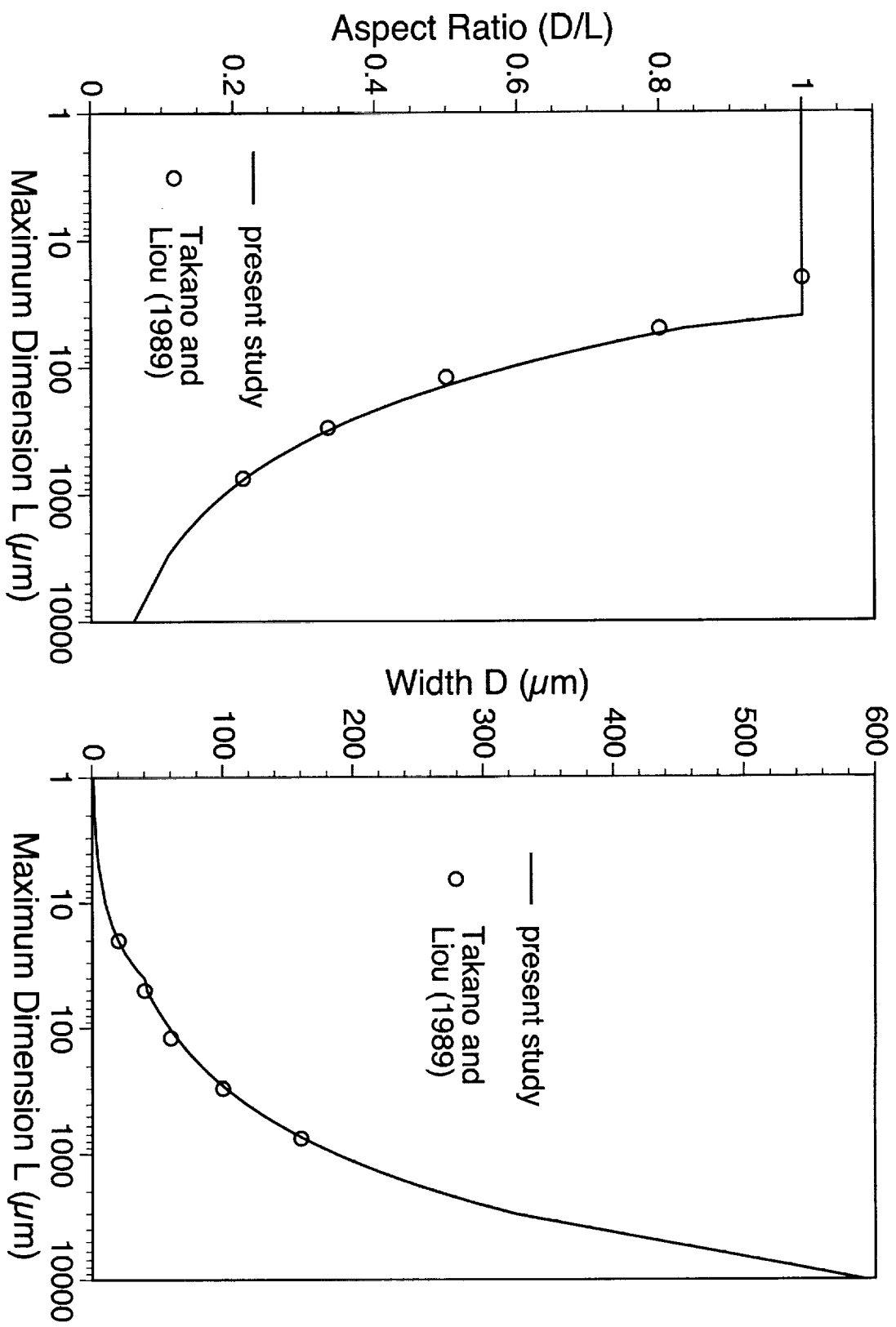


Fig. 1

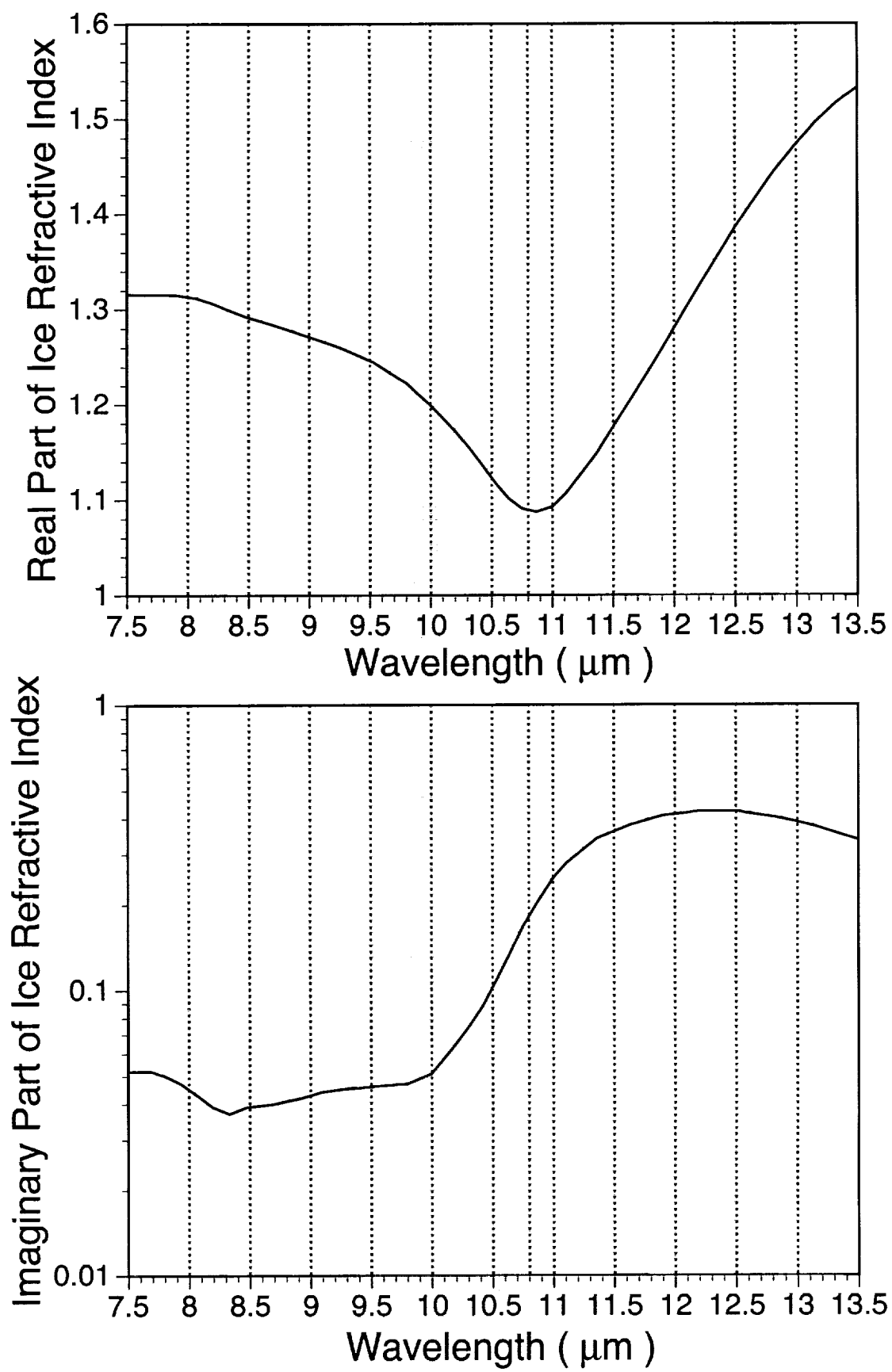


Fig.2



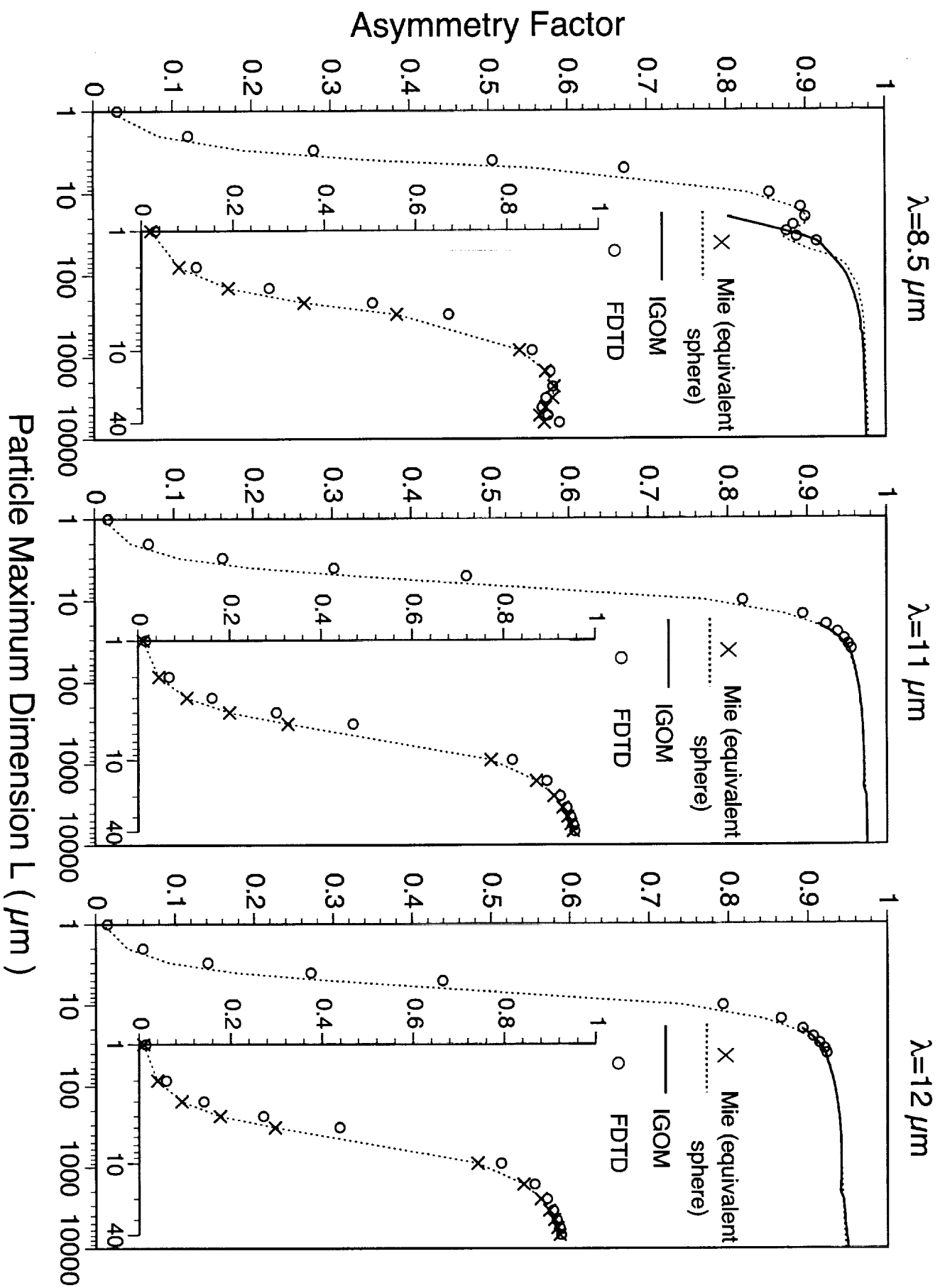


Fig.3

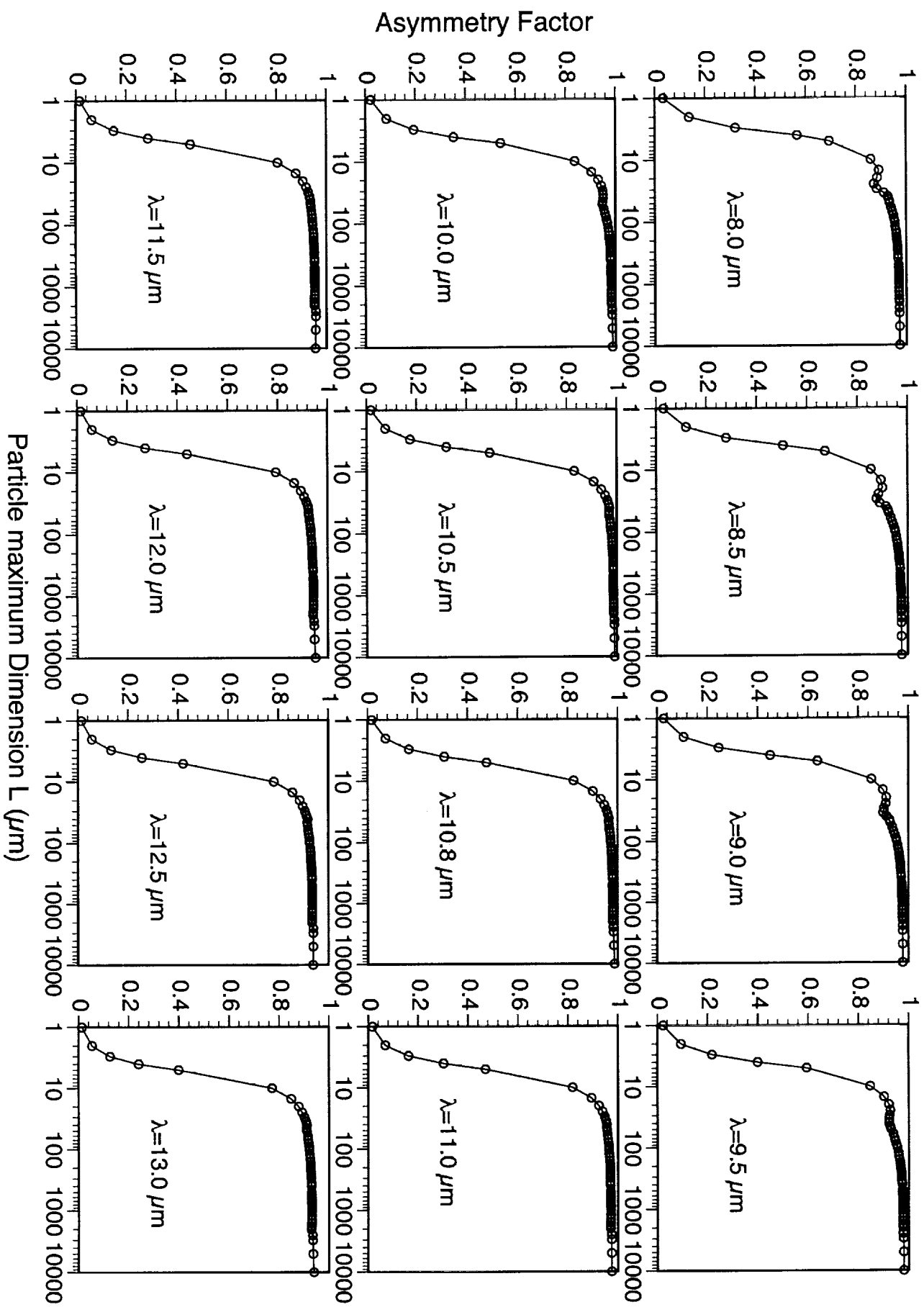
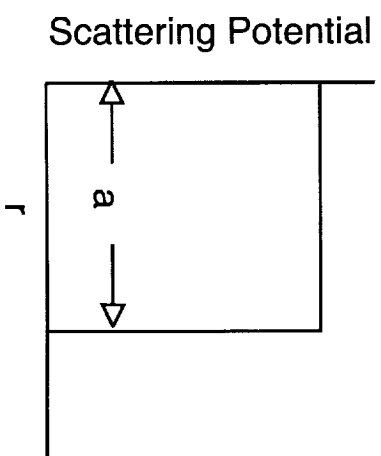
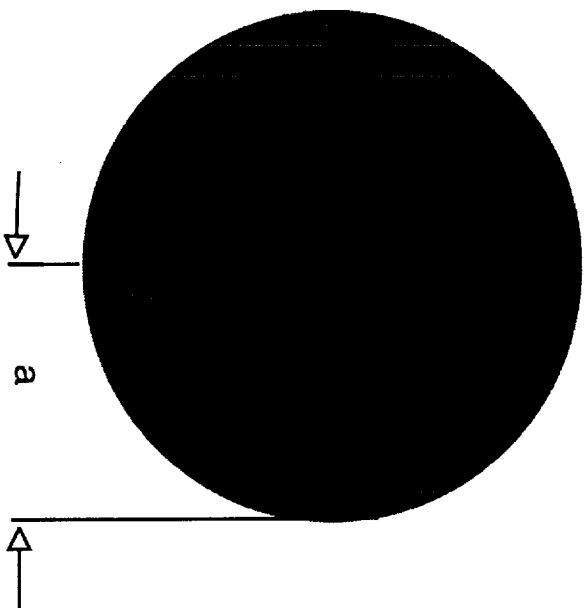


Fig.4

(a)

Conventional non-zero effect region



(b)

Stretched non-zero effect region

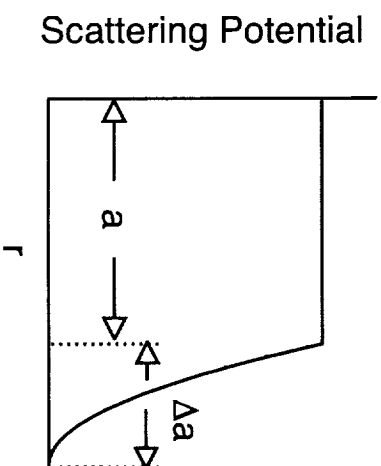
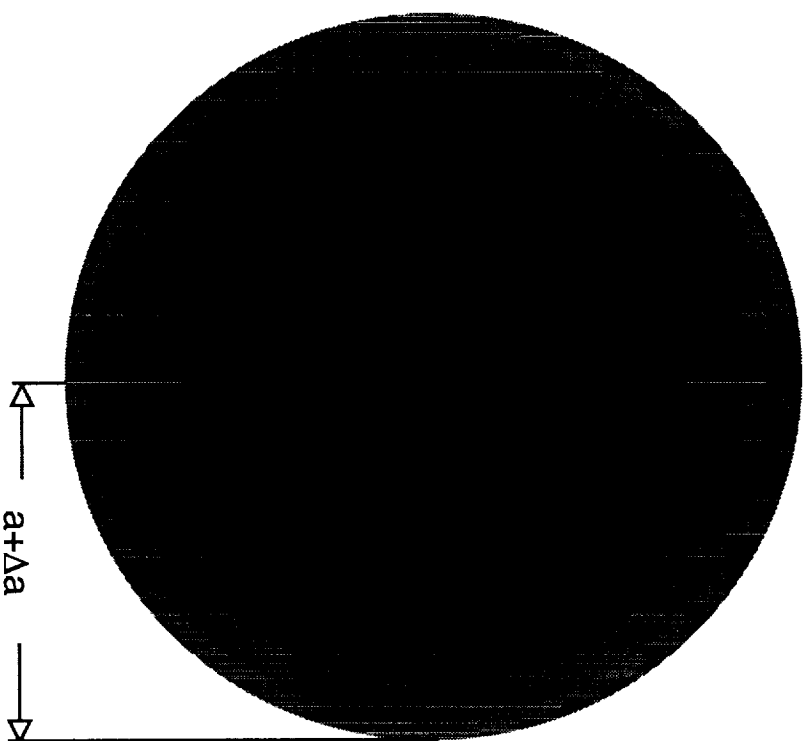


Fig.5

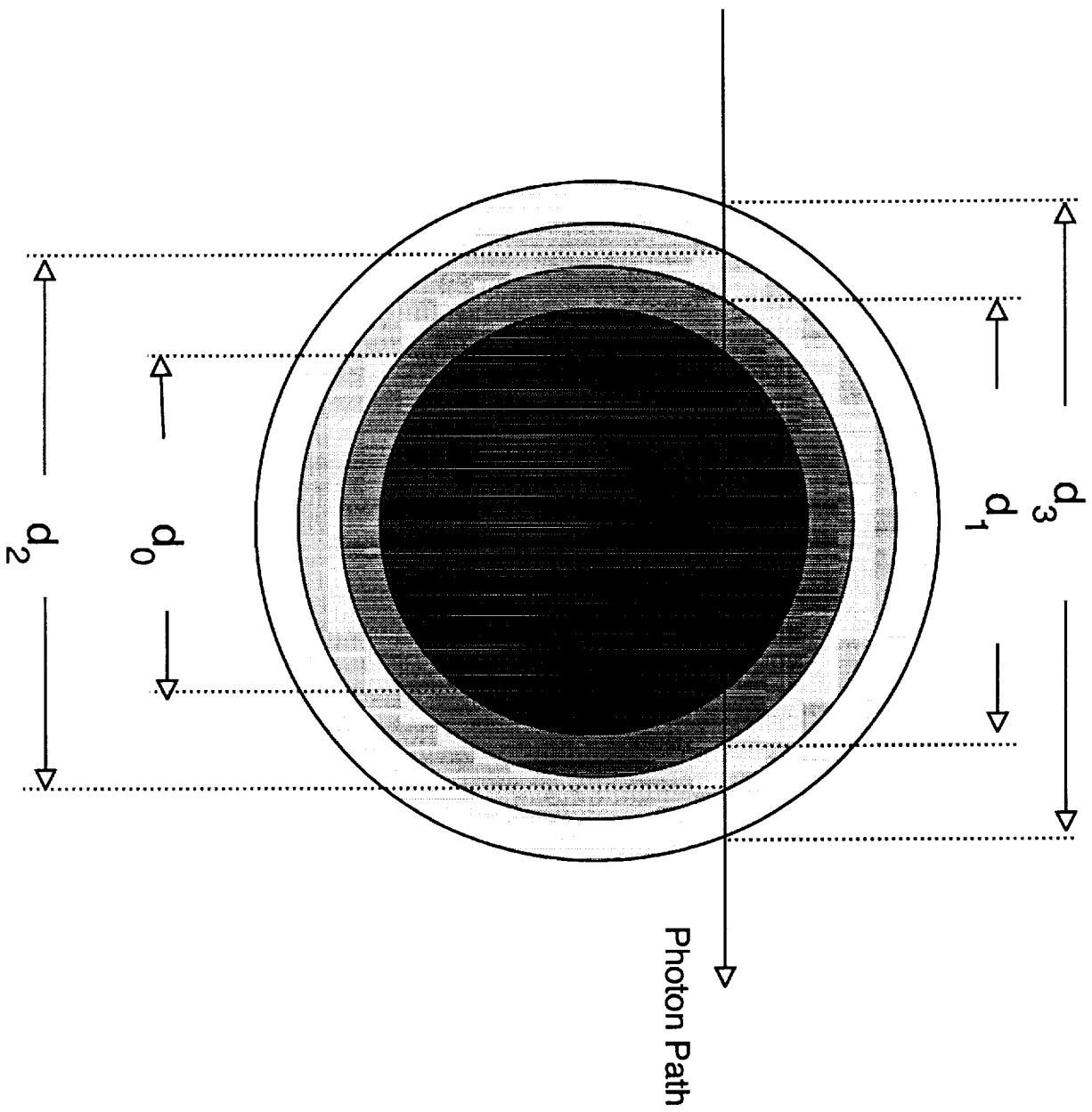


Fig.6

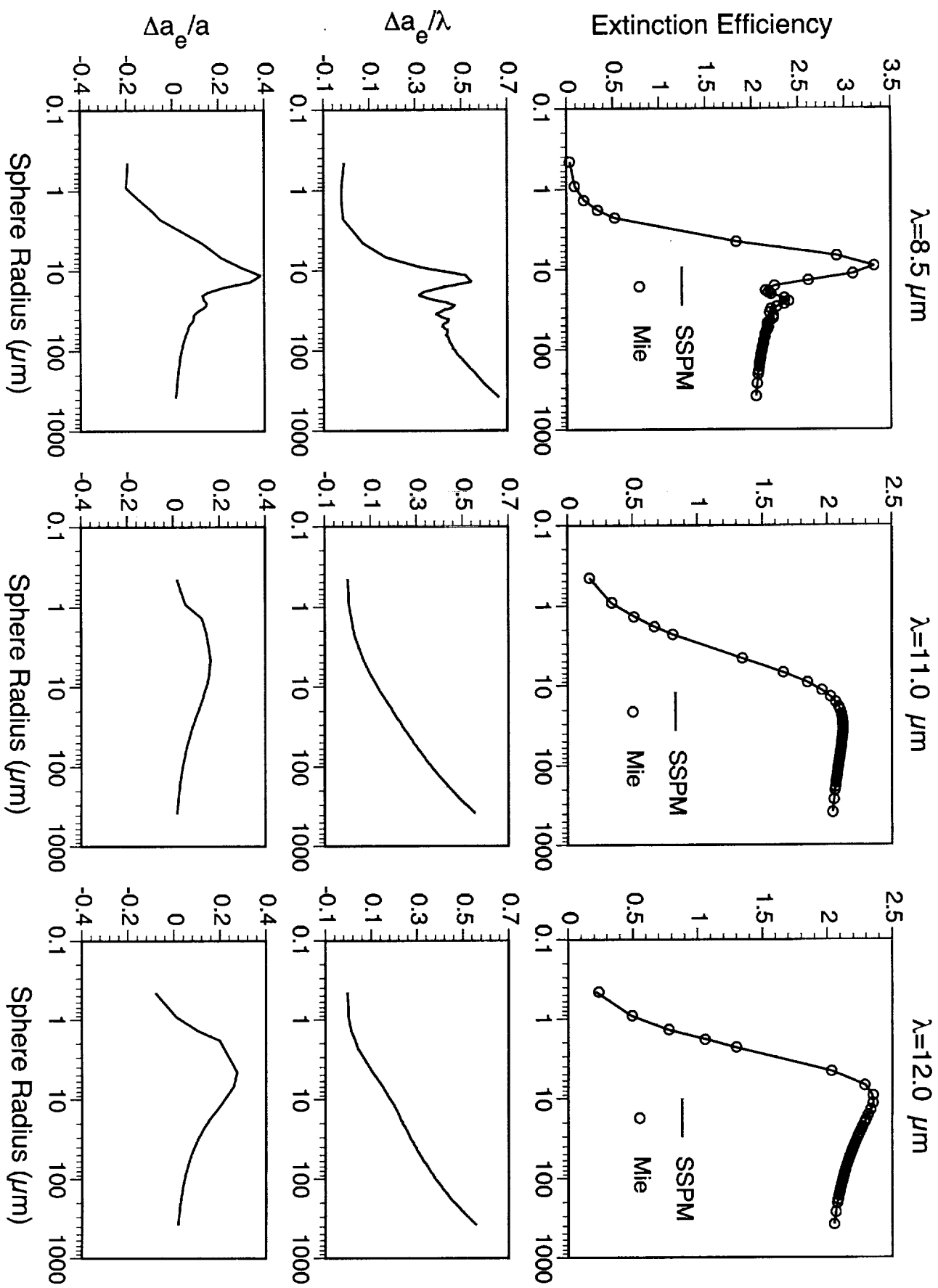


Fig. 7

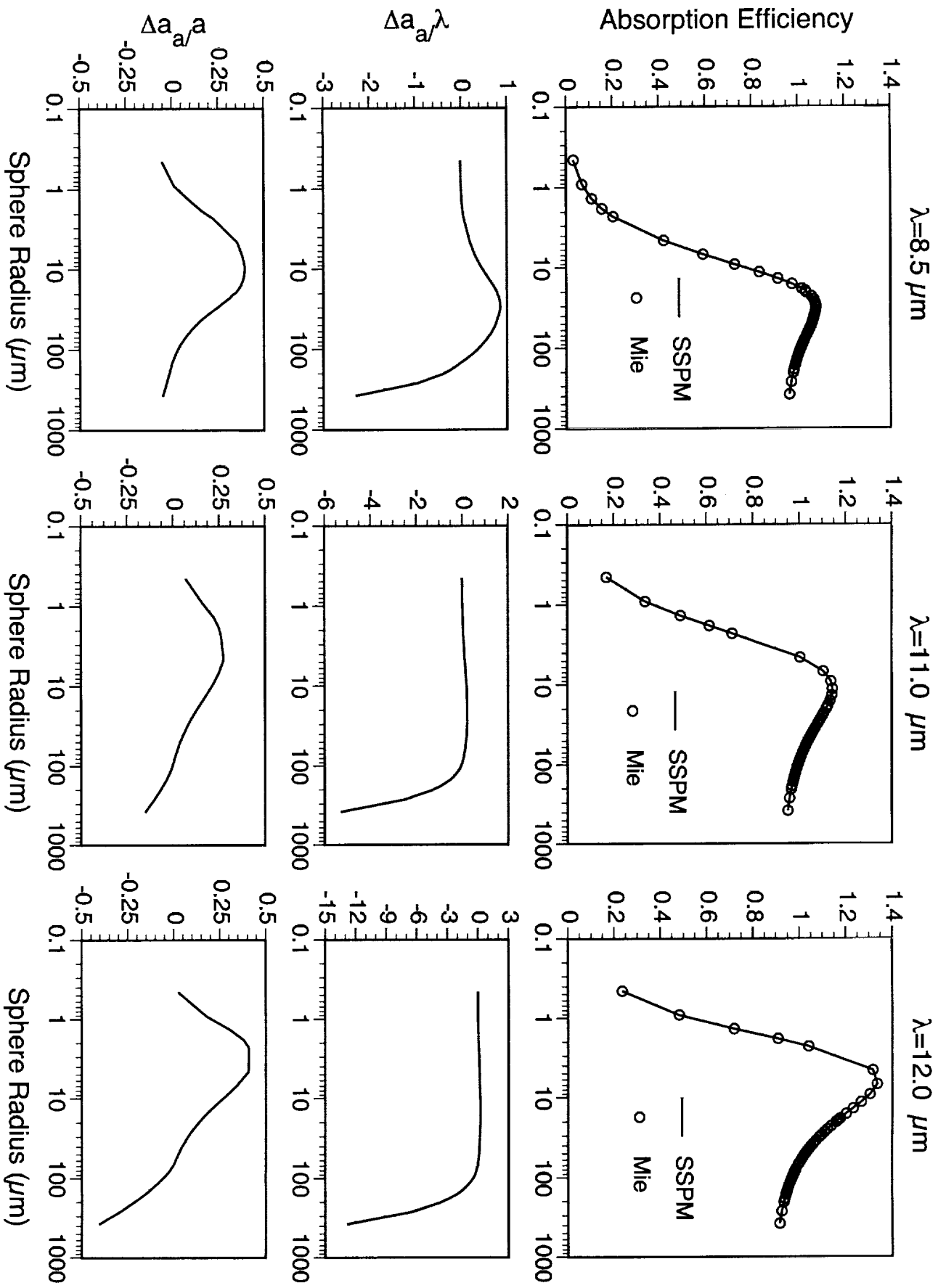


Fig.8

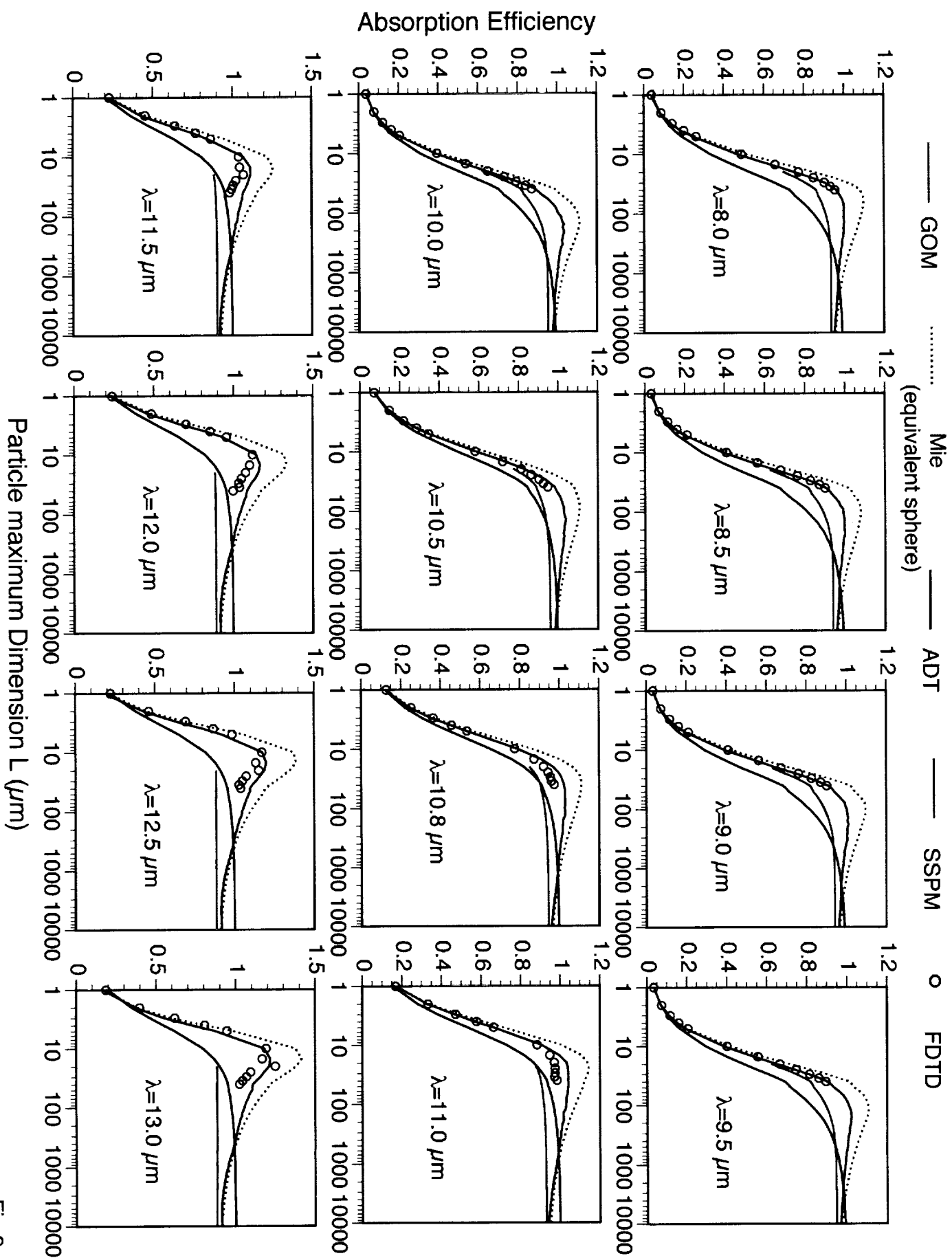


Fig.9

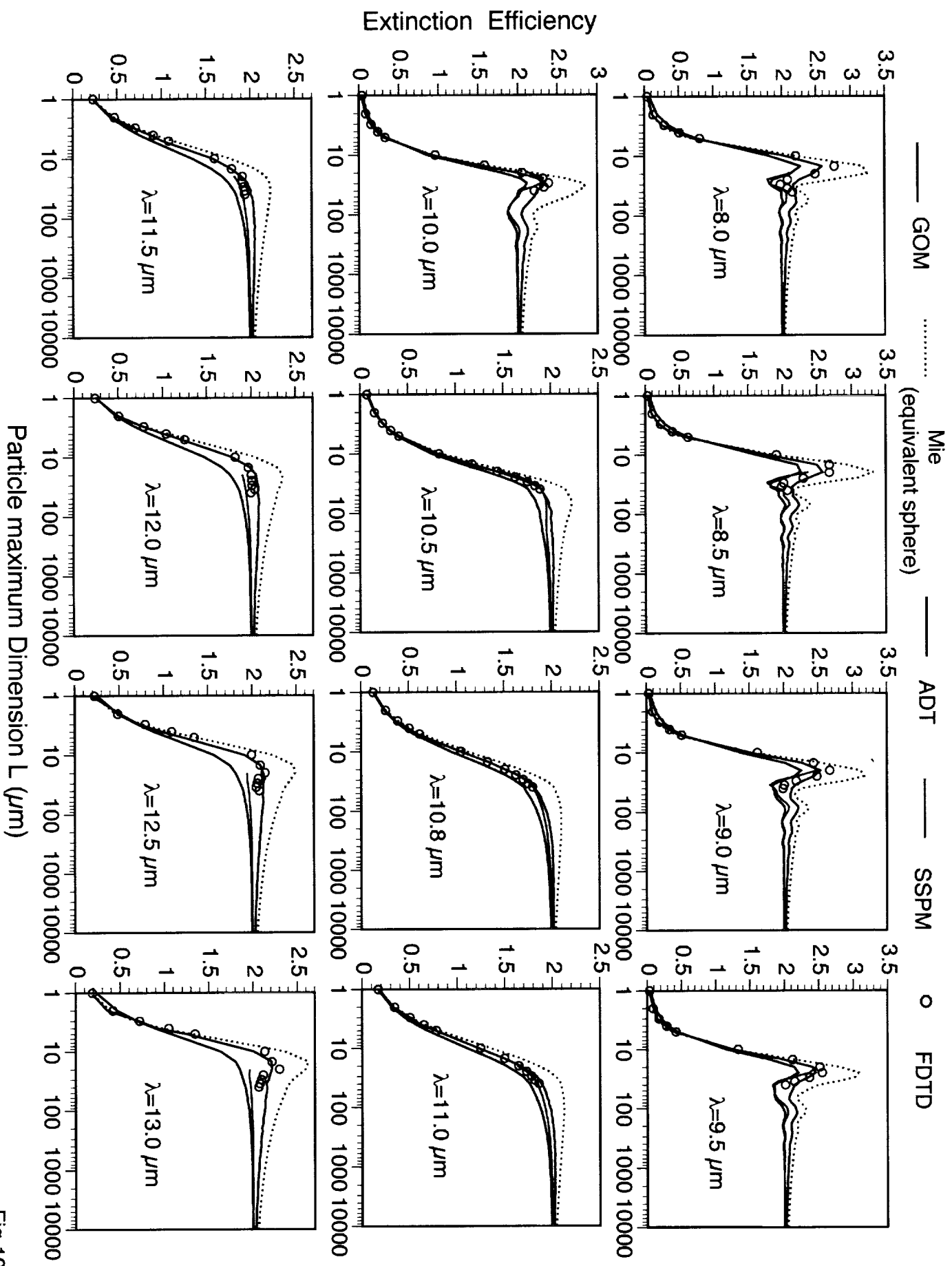


Fig.10



# Extinction (Absorption) Efficiency

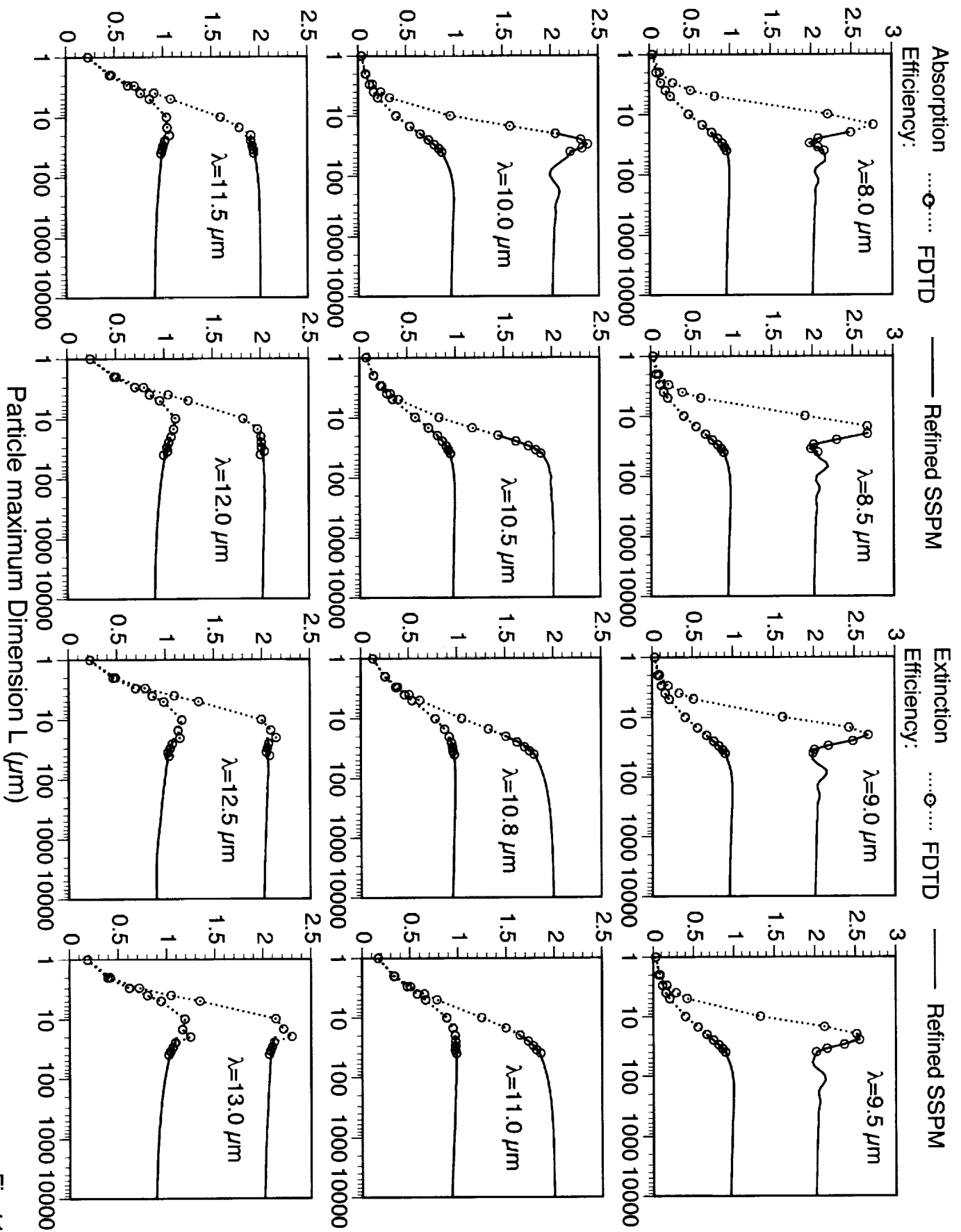


Fig. 11

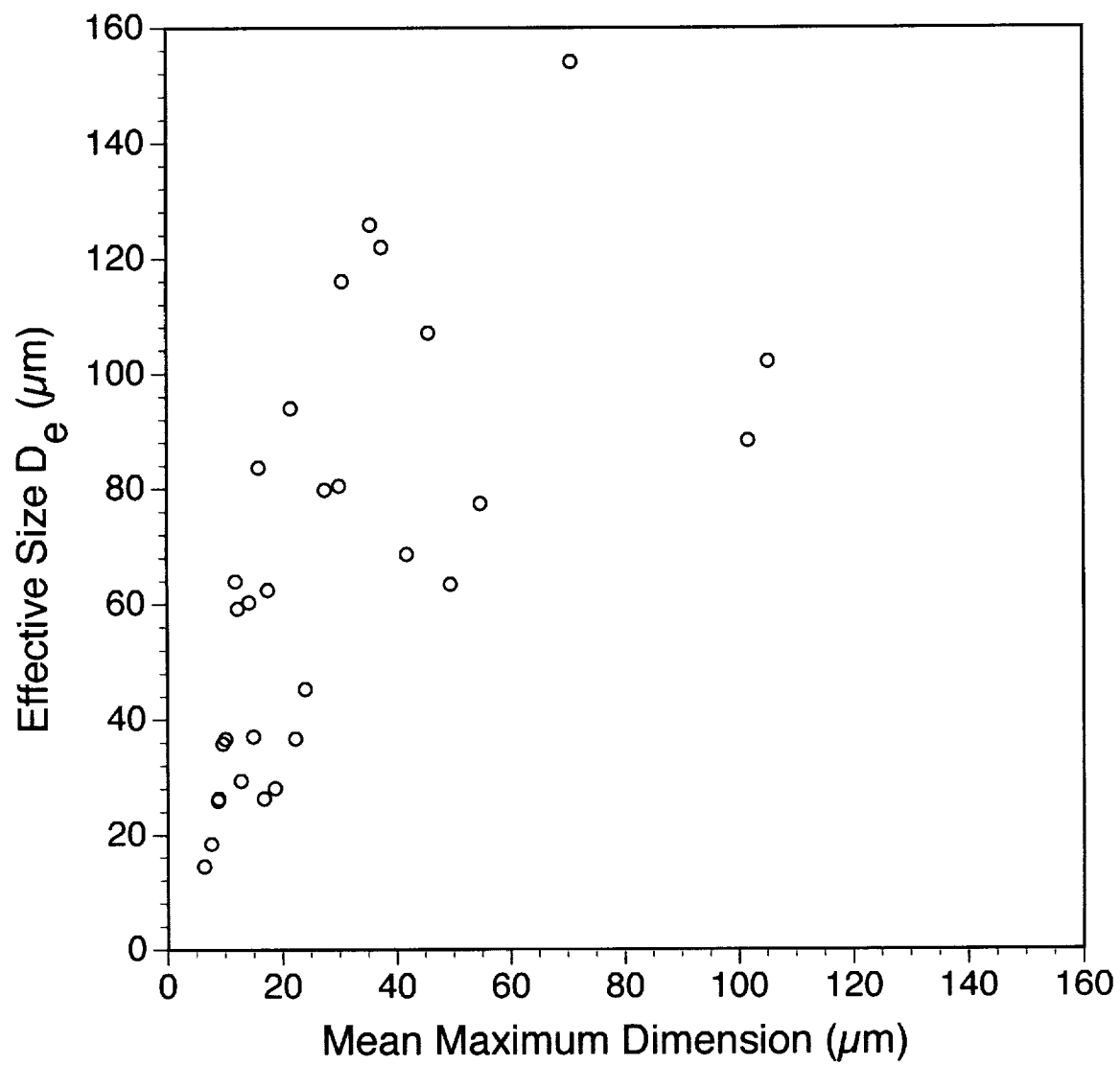


Fig.12

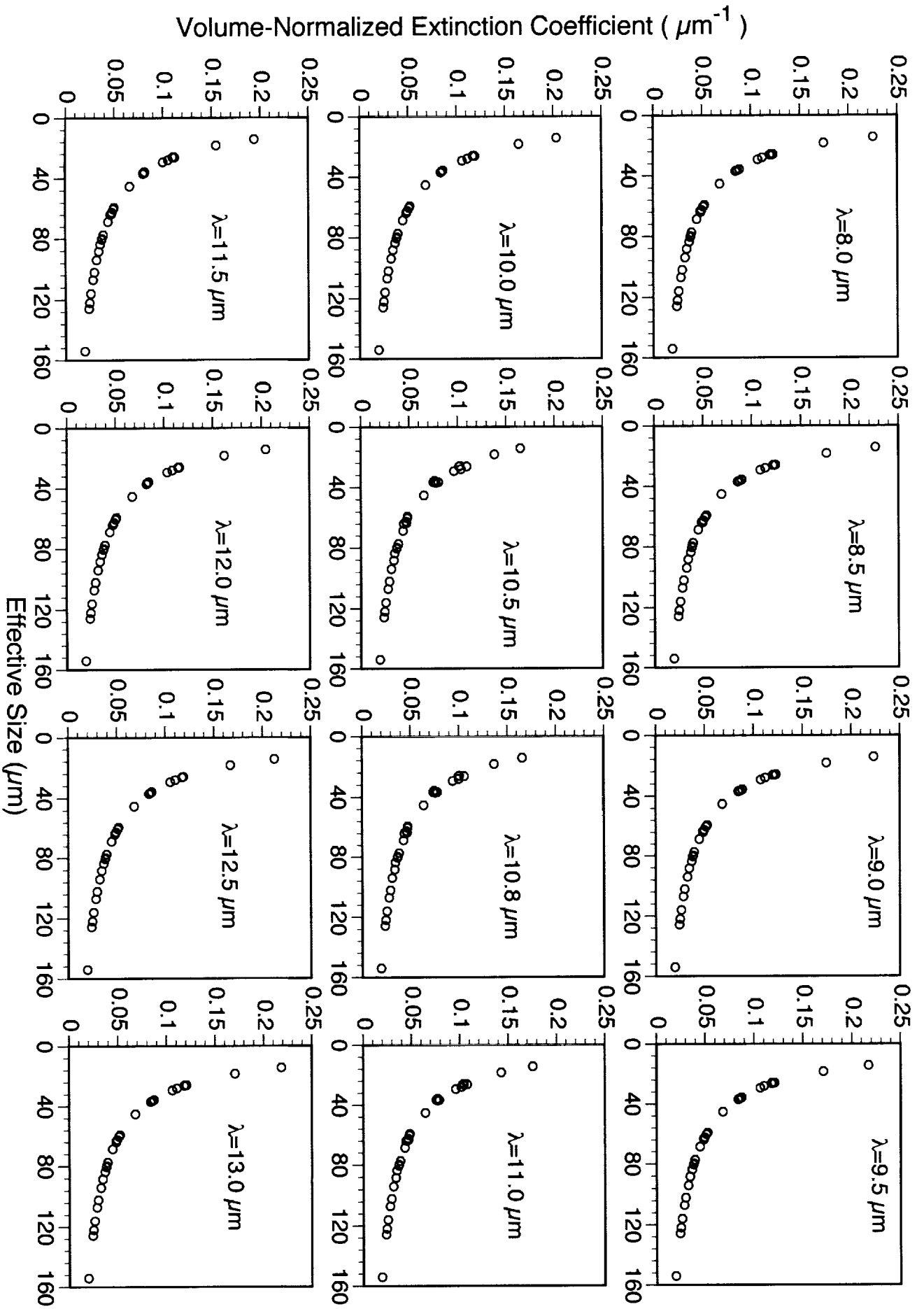


Fig. 13

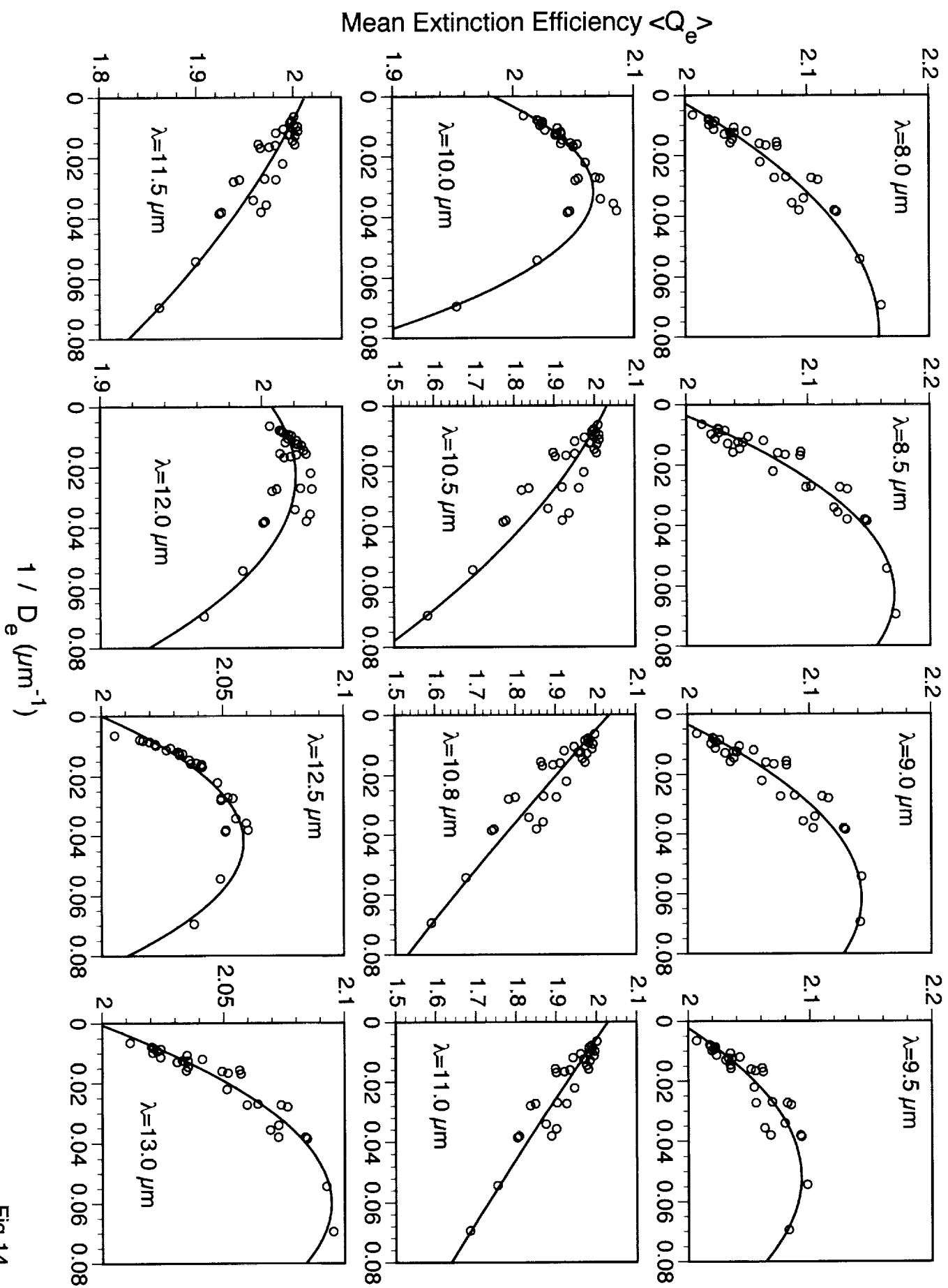


Fig. 14

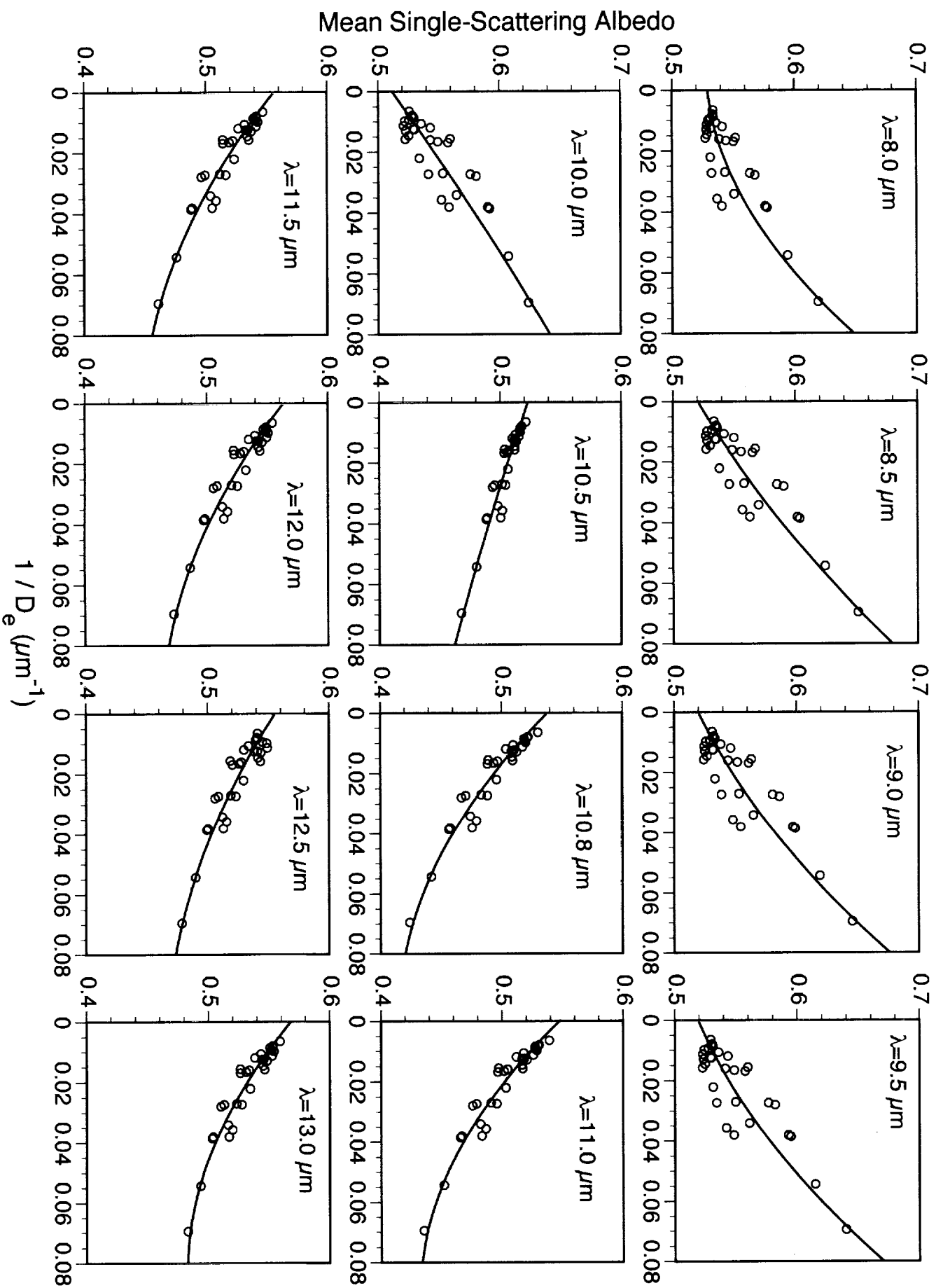


Fig. 15

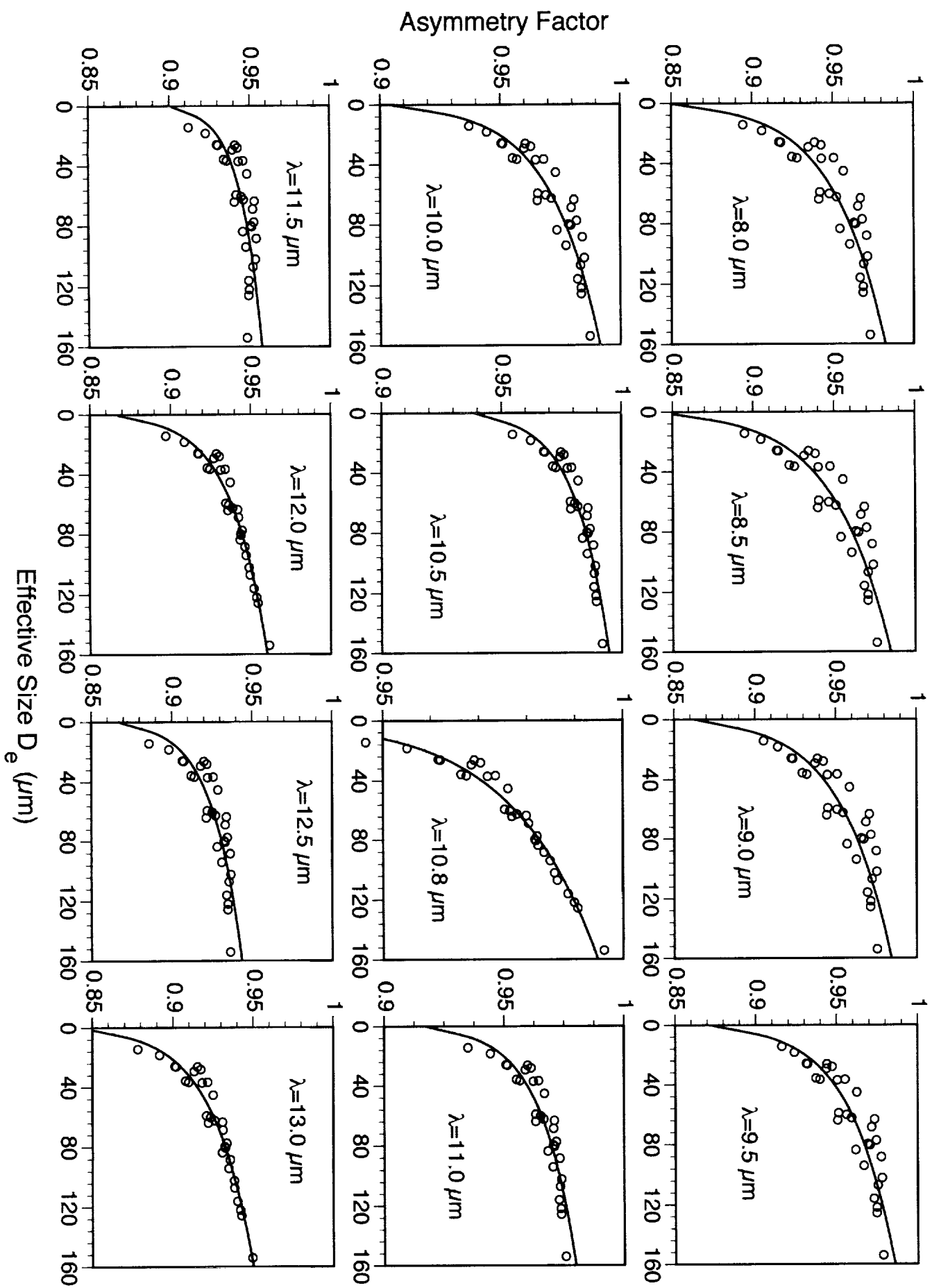


Fig.16